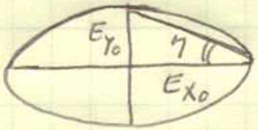
3) a) $\vec{\sigma} = \hat{z}$ ← Homogeneous plane-wave must have real $\vec{\sigma}$, with length 1.

$$\vec{\sigma}\cdot\vec{E}_0 = 0 \Rightarrow \sigma_x E_{x0} + \sigma_y E_{y0} + \sigma_z E_{z0} = 0 \Rightarrow E_{z0} = 0 \Rightarrow \vec{E}_0 = E_{x0}\hat{x} + E_{y0}\hat{y}$$

$$Z_0\vec{H}_0 = \vec{\sigma}\times\vec{E}_0 = \hat{z}\times(E_{x0}\hat{x} + E_{y0}\hat{y}) = E_{x0}\hat{y} - E_{y0}\hat{x} \Rightarrow \vec{H}_0 = \frac{1}{Z_0}(E_{x0}\hat{y} - E_{y0}\hat{x})$$

b) E_{x0} and E_{y0} must be "in phase" for the beam to be linearly polarized; in other words, if $E_{x0} = |E_{x0}|e^{i\phi_{x0}}$ and $E_{y0} = |E_{y0}|e^{i\phi_{y0}}$, then the condition for linear polarization is $\phi_{x0} = \phi_{y0}$.

c) Let $E_{x0} = |E_{x0}|e^{i\phi_{x0}}$ and $E_{y0} = |E_{y0}|e^{i\phi_{y0}}$. The condition for circular polarization is: $|E_{x0}| = |E_{y0}|$ and $\phi_{x0} - \phi_{y0} = \pm 90°$. In other words, $E_{x0} = \pm i E_{y0}$.

d) 

$\tan\eta = \frac{|E_{y0}|}{|E_{x0}|}$ → polarization ellipticity $\eta = \tan^{-1}\frac{|E_{y0}|}{|E_{x0}|}$

e) $$\langle \vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*) = \frac{1}{2}\mathrm{Re}\left\{\vec{E}_0 \cancel{e^{ik_0(\vec{\sigma}\cdot\vec{r}-ct)}} \times \vec{H}_0^* \cancel{e^{-ik_0(\vec{\sigma}\cdot\vec{r}-ct)}}\right\}$$

$$= \frac{1}{2Z_0}\mathrm{Re}\left\{(E_{x0}\hat{x} + E_{y0}\hat{y})\times(-E_{y0}^*\hat{x} + E_{x0}^*\hat{y})\right\} = \frac{1}{2Z_0}\left(|E_{x0}|^2 + |E_{y0}|^2\right)\hat{z}$$

1) a) Incident beam: $\vec{\sigma} = (0,0,-1)$. $Z_0 \vec{H}_0 = \vec{\sigma} \times \vec{E}_0 = -\hat{z} \times E_{ox}\hat{x} = -E_{ox}\hat{y}$

$\langle S_z \rangle = \frac{1}{2} Re(E_x H_y^*) = -\frac{1}{2Z_0} |E_{ox}|^2$ ← rate of flow of optical energy/Unit area in the incident beam (downward).

b)

Reflected beam: $\vec{\sigma}' = (0,0,+1)$, $Z_0 \vec{H}_0' = \vec{\sigma}' \times r\vec{E}_0 = \hat{z} \times r E_{ox}\hat{x} = r E_{ox}\hat{y}$

$\langle S_z' \rangle = \frac{1}{2} Re(E_x' H_y'^*) = \frac{1}{2} Re\left(r E_{ox} \frac{r^* E_{ox}^*}{Z_0}\right) = \frac{1}{2Z_0} |r|^2 |E_{ox}|^2 = \frac{R}{2Z_0}|E_{ox}|^2$

Transmitted beam: $\vec{\sigma}'' = (0,0,-1)$, $Z_0 \vec{H}_0'' = \vec{\sigma}'' \times \tau \vec{E}_0 = -\hat{z} \times \tau E_{ox}\hat{x} = -\tau E_{ox}\hat{y}$

$\langle S_z'' \rangle = \frac{1}{2} Re(E_x'' H_y''^*) = -\frac{1}{2} Re\left(\tau E_{ox} \frac{\tau^* E_{ox}^*}{Z_0}\right) = -\frac{1}{2Z_0} |\tau|^2 |E_{ox}|^2 = -\frac{T}{2Z_0}|E_{ox}|^2$

c) Since n_0 is real (i.e., slab is transparent), the fraction of incident energy that is reflected (i.e., R) plus the fraction that is transmitted (T) must equal unity. Therefore, $R + T = 1$

d) Momentum density $= \frac{\langle S_z \rangle}{c^2}\hat{z}$

In a short time Δt, the light travels a distance of $c\Delta t$ in free-space. With a unit-area cross-section, the corresponding volume is $c\Delta t$. Thus the momentum content of the volume is $\frac{\langle S_z \rangle}{c}\Delta t\,\hat{z}$. The force per unit area, $\hat{\vec{F}}$, is given by:

$$\hat{\vec{F}} = -\frac{\Delta \vec{P}}{\Delta t} = \frac{-1}{\Delta t}\left\{ \langle S_z'' \rangle \frac{\Delta t}{c}\hat{z} + \langle S_z' \rangle \frac{\Delta t}{c}\hat{z} - \langle S_z \rangle \frac{\Delta t}{c}\hat{z} \right\} = -\frac{\langle S_z \rangle}{c}(T - R - 1)\hat{z}$$

$$= \frac{2R}{c}\langle S_z \rangle \hat{z} = -\frac{2R}{2cZ_0}|E_{xo}|^2\hat{z} \Rightarrow \hat{\vec{F}} = -\epsilon_0 R |E_{xo}|^2 \hat{z}.$$