Problem 1)

a) Draw a thin pillbox at the interface, with the top facet of the pill-box above, and its bottom facet below, the interface. The surface integral corresponding to $\vec{\nabla}\cdot\vec{D}$ is $\oint \vec{D}\cdot d\vec{s} = [\vec{D}_\perp(\vec{r}^+,t) - \vec{D}_\perp(\vec{r}^-,t)]\,\Delta s$, with Δs being the surface area of the top (or bottom) facet. The total charge contained within the pill-box is $\sigma_{s\text{-free}}(\vec{r},t)\Delta s$. Setting these equal to each other, we find: $D_\perp(\vec{r}^+,t) - D_\perp(\vec{r}^-,t) = \sigma_{s\text{-free}}(\vec{r},t)$. ✓

b) Draw a thin, rectangular loop at the interface. The short legs of the loop are ⊥ to the interface. The long legs are parallel to the interface, one being above, the other one below. The integral of $\partial\vec{D}/\partial t$ over the area of the loop may be ignored (because the loop is arbitrarily thin).

We distinguish two cases;

Case I: Area of the loop is <u>parallel</u> to $\vec{J}_{s\text{-free}}(\vec{r},t)$ at the location of interest. In this case, the surface current makes no contribution to the integral over the loop surface. We then have $\oint \vec{H}\cdot d\vec{\ell} = [\vec{H}_\parallel(\vec{r}^+,t) - \vec{H}_\parallel(\vec{r}^-,t)]\cdot\vec{\Delta\ell} = 0$
$\Rightarrow \vec{H}_\parallel(\vec{r}^+,t) = \vec{H}_\parallel(\vec{r}^-,t)$. In words, the component of the $\vec{H}$ field that is parallel to the interface <u>AND</u> also parallel to $\vec{J}_{s\text{-free}}(\vec{r},t)$ is continuous across the interface.

Case II: area of the loop is ⊥ to the direction of $\vec{J}_{s\text{-free}}(\vec{r},t)$. The contribution of the surface current to the integral over the loop area is $J_{s\text{-free}}\Delta\ell$. Setting this equal to $\oint \vec{H}\cdot d\vec{\ell}$ we find: $H_\parallel(\vec{r}^+,t) - H_\parallel(\vec{r}^-,t) = J_{s\text{-free}}(\vec{r},t)$. In words, the component of $\vec{H}$ that is parallel to the interface <u>AND</u> perpendicular to $\vec{J}_{s\text{-free}}$ has a <u>discontinuity</u> equal to $J_{s\text{-free}}$ at the interface.

c) Same method as part (b) yields $\vec{E}_\parallel(\vec{r}^+,t) = \vec{E}_\parallel(\vec{r}^-,t)$.

d) Same method as part (a) yields $\vec{B}_\perp(\vec{r}^+,t) = B_\perp(\vec{r}^-,t)$.

Problem 2)

a) $\vec{H}(x,t) = H_0 \cos\{\omega[t - n(\omega)x/c]\}\hat{z}$

Maxwell's Third equation $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial E_y}{\partial x}\hat{z} = -\mu_0 \frac{\partial H_z}{\partial t}\hat{z} \Rightarrow$

$E_0 \omega n(\omega)/c \sin\{\omega[t - n(\omega)x/c]\} = \mu_0 H_0 \omega \sin\{\omega[t - n(\omega)x/c]\} \Rightarrow$

$$H_0 = \frac{n(\omega)}{\mu_0 c} E_0 = \frac{n(\omega)}{Z_0} E_0$$ ✓

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b) $\vec{S}(x,t) = \vec{E}(x,t)\times\vec{H}(x,t) = E_0 H_0 \cos^2\{\omega[t - n(\omega)x/c]\}\hat{x}$ ✓

$\langle\vec{S}(x,t)\rangle = E_0 H_0 \langle\cos^2\{\omega[t - n(\omega)x/c]\}\rangle\hat{x} = \frac{1}{2}\frac{n(\omega)}{Z_0}E_0^2\hat{x}$ ✓

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c) $\vec{E}_1(x,t) + \vec{E}_2(x,t) = E_0\hat{y}\{\cos\{\omega[t - n(\omega)x/c]\} + \cos\{\omega'[t - n(\omega')x/c]\}\}$

$= 2E_0\hat{y}\cos\{\frac{1}{2}(\omega+\omega')t - \frac{1}{2}\frac{\omega n(\omega) + \omega' n(\omega')}{c}x\}\cos\{\frac{1}{2}(\omega'-\omega)t - \frac{\omega' n(\omega') - \omega n(\omega)}{2c}x\}$

$$\simeq \underbrace{2E_0\hat{y}\cos\{\omega_c[t - n(\omega_c)x/c]\}}_{\text{Carrier:}}\ \underbrace{\cos\{\tfrac{1}{2}\Delta\omega[t - \tfrac{\omega' n(\omega') - \omega n(\omega)}{\omega' - \omega}x/c]\}}_{\text{envelope:}}$$

Phase velocity $= \frac{c}{n(\omega_c)}$ Group velocity $= \frac{c}{\frac{d}{d\omega}[\omega n(\omega)]}$

Of Course, $\frac{d}{d\omega}[\omega n(\omega)] = n(\omega_c) + \omega_c n'(\omega_c)$, when the derivative is evaluated at the Center frequency ω_c.