1) $\vec{E} = \vec{E}_0 e^{i\frac{2\pi}{\lambda_0}(\vec{\sigma}\cdot\vec{r}-ct)}$ $\quad \vec{H} = \vec{H}_0 e^{i\frac{2\pi}{\lambda_0}(\vec{\sigma}\cdot\vec{r}-ct)}$ $\quad \checkmark \begin{cases} \vec{\nabla}\cdot\vec{E}=0 \Rightarrow \vec{\sigma}\cdot\vec{E}_0=0 \\ \vec{\nabla}\cdot\vec{B}=0 \Rightarrow \vec{\sigma}\cdot\vec{H}_0=0 \end{cases}$

$\checkmark\ \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow i\frac{2\pi}{\lambda_0}\vec{\sigma}\times\vec{E}_0 = -(-i\frac{2\pi c}{\lambda_0})\mu_0\vec{H}_0 \Rightarrow \vec{\sigma}\times\vec{E}_0 = Z_0\vec{H}_0$

$\checkmark\ \vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \Rightarrow i\frac{2\pi}{\lambda_0}\vec{\sigma}\times\vec{H}_0 = -i\frac{2\pi c}{\lambda_0}\epsilon_0\epsilon\vec{E}_0 \Rightarrow \vec{\sigma}\times\vec{H}_0 = -\frac{\epsilon}{Z_0}\vec{E}_0 \Rightarrow \vec{\sigma}\times(\vec{\sigma}\times\vec{E}_0) = -\epsilon\vec{E}_0$

$\Rightarrow \underbrace{(\vec{\sigma}\cdot\vec{E}_0)}_{0}\vec{\sigma} - (\vec{\sigma}\cdot\vec{\sigma})\vec{E}_0 = -\epsilon\vec{E}_0 \Rightarrow \vec{\sigma}\cdot\vec{\sigma} = \epsilon = n^2$

a) In free space $n=1 \Rightarrow \vec{\sigma} = \hat{z} \Rightarrow Z_0\vec{H}_i = \hat{z}\times E_i\hat{x} \Rightarrow Z_0\vec{H}_i = E_i\hat{y}$

b) Inside the slab $\vec{\sigma} = n\hat{z} \Rightarrow Z_0\vec{H}_t = \vec{\sigma}\times\vec{E}_t = n\hat{z}\times\vec{E}_t \Rightarrow Z_0\vec{H}_t = nE_t\hat{y}$

c) Rate of flow of energy: in free space $\langle\vec{S}_i\rangle = \frac{1}{2}\vec{E}_i\times\vec{H}_i = \frac{1}{2Z_0}E_i^2\hat{z}$

" " " " " : in transparent glass $\langle\vec{S}_t\rangle = \frac{1}{2}\vec{E}_t\times\vec{H}_t = \frac{n}{2Z_0}E_t^2\hat{z}$

$\langle\vec{S}_i\rangle = \langle\vec{S}_t\rangle \Rightarrow \frac{1}{2Z_0}E_i^2 = \frac{n}{2Z_0}E_t^2 \Rightarrow E_t = \frac{E_i}{\sqrt{n}}$; $H_t = \frac{nE_t}{Z_0} = \frac{\sqrt{n}E_i}{Z_0}$

d) $\vec{E}$-field energy density inside dispersionless medium $= \frac{1}{2}\epsilon_0\epsilon E_t^2 = \frac{1}{2}\epsilon_0 n^2(\frac{E_i}{\sqrt{n}})^2 = \frac{1}{2}\epsilon_0 n E_i^2$ ✓

$\vec{H}$-field energy density " " " $= \frac{1}{2}\mu_0 H_t^2 = \frac{1}{2}\mu_0(\frac{\sqrt{n}E_i}{Z_0})^2 = \frac{1}{2}\epsilon_0 n E_i^2$ ✓

✓ Therefore, the $\vec{E}$- and $\vec{H}$-field energy densities are equal.

Let the pulse duration be T and its cross-sectional area A. In the free space, the pulse length is cT, its volume is cTA, its energy density is $\frac{1}{2}\epsilon_0 E_i^2 + \frac{1}{2}\mu_0 H_i^2 = \frac{1}{2}\epsilon_0 E_i^2 + \frac{1}{2}\mu_0(\frac{E_i}{Z_0})^2 = \epsilon_0 E_i^2$. Therefore, its total energy (in the free space) is $\epsilon_0 E_i^2 cTA$.

Inside the glass medium, the pulse length is cT/n, the volume is cTA/n, the energy density is $\frac{1}{2}\epsilon_0 n E_i^2 + \frac{1}{2}\epsilon_0 n E_i^2 = \epsilon_0 n E_i^2$. Therefore, the total energy of the pulse (in the glass) is $\epsilon_0 E_i^2 cTA$. The energy is thus conserved.

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2) a) Incident beam: $H_0 = E_0/Z_0$; transmitted beam $H_t = \frac{nE_t}{Z_0} = \frac{n\tau E_0}{Z_0}$;

Reflected beam: $H_r = E_r/Z_0 = \rho E_0/Z_0$.

b) Incident beam: $\langle S_i \rangle = \frac{1}{2}|\vec{E}_0 \times \vec{H}_0| = E_0^2/(2Z_0)$

Reflected beam: $\langle S_r \rangle = \frac{1}{2}|\vec{E}_r \times \vec{H}_r| = \rho^2 E_0^2/(2Z_0)$

Transmitted beam: $\langle S_t \rangle = \frac{1}{2}|\vec{E}_t \times \vec{H}_t| = n\tau^2 E_0^2/(2Z_0)$

c) Cross-sectional areas of the incident and reflected beams are proportional to $\zeta \cos\theta$, while the cross-sectional area of the transmitted beam is proportional to $\zeta \cos\theta'$. From Snell's law, we have $\sin\theta = n \sin\theta' \Rightarrow \cos\theta' = \sqrt{1-\sin^2\theta'} = \sqrt{1-\frac{\sin^2\theta}{n^2}}$. We equate the influx of optical energy to the sum of reflected and transmitted energy flux, and obtain:

$$\frac{1}{\cancel{2Z_0}}\cancel{E_0^2}\cancel{\zeta}\cos\theta = \frac{1}{\cancel{2Z_0}}\rho^2\cancel{E_0^2}\cancel{\zeta}\cos\theta + \frac{1}{\cancel{2Z_0}}n\tau^2\cancel{E_0^2}\cancel{\zeta}\cos\theta' \Rightarrow \rho^2 + \frac{n\cos\theta'}{\cos\theta}\tau^2 = 1 \Rightarrow$$

$$\rho^2 + \frac{\sqrt{n^2-\sin^2\theta}}{\cos\theta}\tau^2 = 1$$

d) <u>None</u> of the relations obtained in (a), (b), (c) will change as a result of switching from p-light to s-light. Therefore, the relationship between ρ and τ, obtained in part (c), will remain the same.