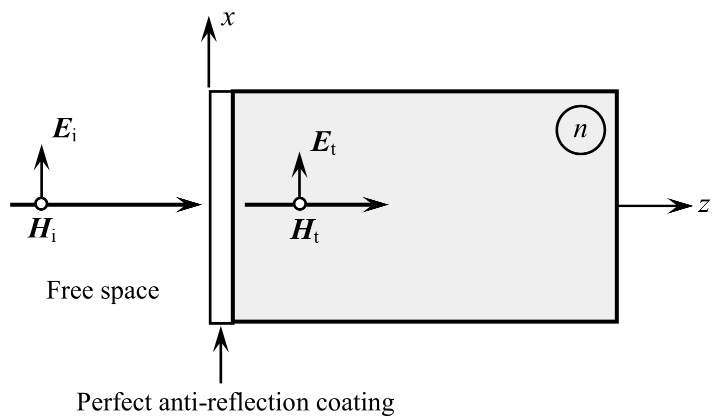
A semi-infinite slab of transparent, non-magnetic glass (refractive index $= n$) has a *perfect* anti-reflection coating applied to its entrance facet. A monochromatic, linearly-polarized plane-wave arrives at the slab at normal incidence, as shown. The incidence medium is free space, the vacuum wavelength of the light is λ_0, and the incident E-field is along the x-axis.

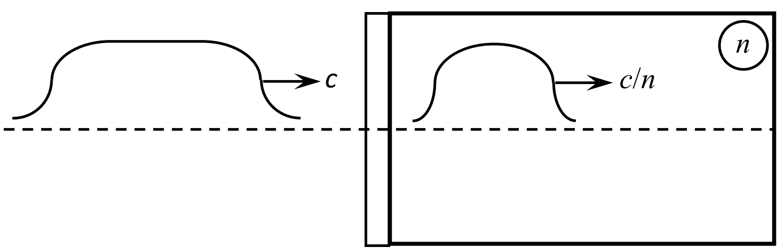
a) What is the relation between the incident E- and H-fields, E_i and H_i, in terms of the impedance of free-space, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$?



b) What is the relation between the fields E_t, H_t transmitted into the slab in terms of Z_0 and n?

c) Without making any assumptions about the structure of the anti-reflection coating, simply knowing that the optical energy of the beam passes entirely from free space into the slab, determine the relation between the incident and transmitted E-fields E_i and E_t.

d) Assume now that, instead of a plane-wave, the incident beam is a pulse of light having the same central wavelength λ_0 as before. Moreover, the front-facet coating is effective as a perfect anti-reflection coating for the entire pulse, and the semi-infinite slab is free from dispersion, so that, inside the slab, the pulse propagates with velocity c/n, as shown. What are the E- and H-field energies inside the slab? Is the total E-field energy of the pulse equal to its total H-field energy? Is the pulse energy conserved before and after incidence?



Hint: You may find the vector identity $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$ useful.

A linearly-polarized, monochromatic plane-wave propagates along the x-axis, with its E-field amplitude given as $\boldsymbol{E}(x,t) = E_0 \cos\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{y}}$. The host medium is a homogeneous, linear, isotropic, non-magnetic (i.e., $\mu(\omega) = 1$), transparent dielectric, whose frequency-dependent refractive index is specified as $n(\omega) = \sqrt{\varepsilon(\omega)}$.

a) Find the magnetic field $\boldsymbol{H}(x,t)$ of the plane-wave in terms of $E_0, c, \omega, n(\omega)$, and the impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

b) Find the Poynting vector $\boldsymbol{S}(x,t)$ of the above plane-wave, then determine the time-averaged rate-of-flow of optical energy (per unit area per unit time) along the x-axis.

c) Assume a second plane-wave, *identical* with the one above *except* for its frequency ω' differing slightly from ω, is co-propagating with the above plane-wave. Write an expression for the combined E-field of the superposed plane-waves. From this expression, identify the carrier and the envelope of the beat waveform. In terms of $c, \omega_c = ½(\omega + \omega'), \Delta\omega = \omega' - \omega$, $n(\omega_c)$ and $\mathrm{d}n(\omega)/\mathrm{d}\omega$, what are the *phase* and *group* velocities of the combined waveform?

Hint: You may find the trigonometric identity $\cos a + \cos b = 2\cos[½(a+b)]\cos[½(a-b)]$ useful.