Summer 2017 Written Comprehensive Exam
Opti 501, day 1

System of units: MKSA (also known as SI)

1) A plane wave having wave-vector $\boldsymbol{k}$ and frequency ω propagates in free space. Let the electric and magnetic fields of the plane-wave be written as

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)],$$
$$\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)].$$

No sources are assumed to reside in free space; therefore, $\rho_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, $\boldsymbol{P}(\boldsymbol{r},t) = 0$, and $\boldsymbol{M}(\boldsymbol{r},t) = 0$.

2 Pts a) What does Maxwell's *first* equation have to say about the relation between $\boldsymbol{E}_0$ and $\boldsymbol{k}$?

2 Pts b) What does Maxwell's *fourth* equation have to say about the relation between $\boldsymbol{H}_0$ and $\boldsymbol{k}$?

2 Pts c) Apply Maxwell's *second* equation to derive a relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

2 Pts d) Apply Maxwell's *third* equation to derive another relation connecting $\boldsymbol{E}_0$, $\boldsymbol{H}_0$, $\boldsymbol{k}$, and ω.

2 Pts e) Combine the results obtained in parts (a)-(d) to eliminate $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ from the equations, thus arriving at the connection between $\boldsymbol{k}$ and ω for plane-waves that travel in free space.

Hint: In part (e), the following vector identity will be helpful: $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$.

Summer 2017 Written Comprehensive Exam
Opti 501, day 2

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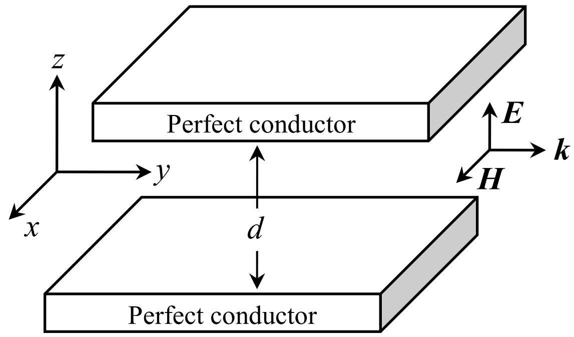
2) In the free-space region between two infinitely large, perfectly conducting parallel plates, an electromagnetic plane-wave propagates along the y-axis, as shown. The electric and magnetic fields of the plane-wave are given by

$$\boldsymbol{E}(\boldsymbol{r},t) = E_o \cos(k_o y - \omega_o t)\hat{\boldsymbol{z}},$$

$$\boldsymbol{H}(\boldsymbol{r},t) = H_o \cos(k_o y - \omega_o t)\hat{\boldsymbol{x}},$$



where E_0 and H_0 are the field amplitudes, k_o is the propagation constant (also known as the wave-number), and ω_0 is the angular frequency of the plane-wave. All the above parameters, i.e., E_o, H_0, k_o and ω_0, are real-valued constants.

4 Pts a) Write Maxwell's equations in the region between the plates (i.e., $-½d < z < ½d$), then find the relationship between k_o and ω_o on the one hand, and that between E_o and H_o on the other.

Hint: These relations should involve the speed of light in vacuum, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and the impedance of free space, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

4 Pts b) Considering that the E and H fields inside the (perfectly conducting) plates must be zero, use Maxwell's boundary conditions to find the surface charge-density $\sigma_s(x, y, z = \pm½d, t)$ and the surface current-density $\boldsymbol{J}_s(x, y, z = \pm½d, t)$ on the inner surfaces of both plates.

2 Pts c) Show that the surface charge and current densities obtained in part (b) satisfy the charge-current continuity equation.