Summer 2016 Written Comprehensive Exam
Opti 501

System of units: MKSA

A monochromatic plane electromagnetic wave propagates in free space along the z-axis. The beam is linearly polarized along the x-axis, having E-field amplitude $E_0\hat{\boldsymbol{x}}$ and H-field amplitude $H_0\hat{\boldsymbol{y}}$. In general, the field amplitudes are complex-valued; for instance, $E_0 = |E_0|\exp(\mathrm{i}\varphi_0)$.

3 Pts a) In terms of the frequency ω_0 of the oscillations, the speed c of light in vacuum, the impedance Z_0 of free space, and the E-field amplitude and phase, $|E_0|$ and φ_0, write expressions for the *real-valued* $\boldsymbol{E}$ and $\boldsymbol{H}$ fields as functions of the space-time coordinates (x, y, z, t).

3 Pts b) Determine the rate of flow of electromagnetic energy (per unit cross-sectional area per unit time) at a given instant of time, say, $t = t_0$, at two points P_1 and P_2 located at $(x, y, z) = (0, 0, z_1)$ and $(x, y, z) = (0, 0, z_2)$, where z_1 and z_2 are two arbitrary points along the z-axis.

4 Pts c) According to your answer to part (b), the rate of flow of energy at (P_1, t_0) differs from that at (P_2, t_0) —unless the distance $z_2 - z_1$ happens to be a half-integer-multiple of the wavelength $\lambda_0 = 2\pi c/\omega$. Identify where the missing energy has gone. (You must provide a detailed answer that accounts for the missing energy with perfect accuracy.)

Hint: The following identities will be helpful:

$$\cos^2 x = ½(1 + \cos 2x)$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right); \qquad \sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right); \qquad \cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

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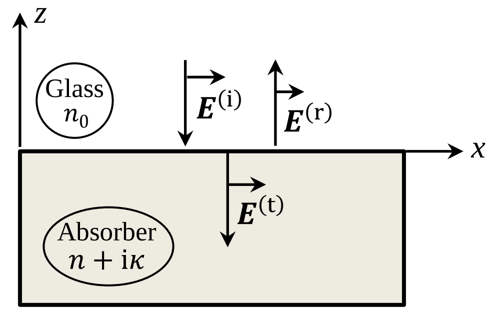
A plane electromagnetic wave of frequency ω arrives at normal incidence at the interface between a transparent dielectric medium of refractive index n_0 and an absorptive medium specified by its complex refractive index $n + \mathrm{i}\kappa$. The incident beam is linearly polarized, having E-field amplitude $E_0\hat{\boldsymbol{x}}$ and H-field amplitude $H_0\hat{\boldsymbol{y}}$. Assume $\mu(\omega) = 1$ for both media, the speed of light in vacuum is $c = 1/\sqrt{\mu_0\varepsilon_0}$, and the impedance of free space is $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.



2 Pts a) Write expressions for the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields in the semi-infinite medium of incidence (transparent dielectric), and in the semi-infinite absorptive substrate.

2 Pts b) Match the boundary conditions at the interface between the two media (located at $z = 0$), then proceed to determine the Fresnel reflection and transmission coefficients at the boundary.

3 Pts c) Use the *time-averaged* Poynting vector $\langle \boldsymbol{S}(\boldsymbol{r}, t)\rangle$ to determine the rate of flow of electromagnetic energy (per unit cross-sectional area, per unit time) in the incident, reflected, and transmitted beams at the interface between the two media (i.e., at $z = 0^+$ and $z = 0^-$).

3 Pts d) Confirm that the difference between the incident and reflected optical energies is in fact equal to the energy captured within the absorptive substrate.