Spring 2016 Written Comprehensive Exam
Opti 501

System of units: MKSA

In electromagnetic theory, a wave equation is a 2nd order partial differential equation that is satisfied by a single field, such as the scalar potential $\psi(\boldsymbol{r},t)$, the vector potential $\boldsymbol{A}(\boldsymbol{r},t)$, the electric field $\boldsymbol{E}(\boldsymbol{r},t)$, or the magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$. Examples of the wave equation (in the Lorenz gauge) are

$$\boldsymbol{\nabla}^2\psi(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2\psi(\boldsymbol{r},t)}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho_{\text{total}}(\boldsymbol{r},t).$$

$$\boldsymbol{\nabla}^2\boldsymbol{A}(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}(\boldsymbol{r},t)}{\partial t^2} = -\mu_0\boldsymbol{J}_{\text{total}}(\boldsymbol{r},t).$$

In this problem you are asked to derive a wave equation for the electric field, and another wave equation for the magnetic field, under certain special circumstances. Limiting the time-dependence of the fields to single-frequency (i.e., monochromatic) behavior, the fields will be written as $\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)$ and $\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)$. Consequently, the desired wave equations will be 2nd order partial differential equations in spatial coordinates only — one satisfied by $\boldsymbol{E}(\boldsymbol{r})$, the other by $\boldsymbol{H}(\boldsymbol{r})$. It will be further assumed that free charge and free current are absent from the system, that is, $\rho_{\text{free}}(\boldsymbol{r},t) = 0$ and $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) = 0$, and that the electro-magnetic fields reside in a homogeneous, linear, isotropic medium specified by its permittivity $\varepsilon_0\varepsilon(\omega)$ and permeability $\mu_0\mu(\omega)$.

4 Pts a) Write Maxwell's equations for the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields, given the above restrictions.

4 Pts b) Eliminate the $\boldsymbol{H}$ field from Maxwell's equations in order to arrive at the wave equation satisfied by $\boldsymbol{E}(\boldsymbol{r})$.

2 Pts c) Eliminate the $\boldsymbol{E}$ field from Maxwell's equations in order to arrive at the wave equation satisfied by $\boldsymbol{H}(\boldsymbol{r})$.

Hint: The identity $\boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times\boldsymbol{F}(\boldsymbol{r})] = \boldsymbol{\nabla}[\boldsymbol{\nabla}\cdot\boldsymbol{F}(\boldsymbol{r})] - \boldsymbol{\nabla}^2\boldsymbol{F}(\boldsymbol{r})$ is valid for any vector field $\boldsymbol{F}(\boldsymbol{r})$ residing in 3-dimensional space.

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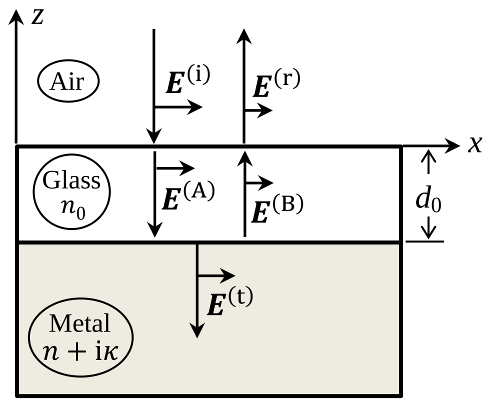

A smooth and flat metallic surface is coated with a thin dielectric layer of thickness d_0 and real-valued refractive index n_0. The complex refractive index of the metal is $\sqrt{\mu(\omega)\varepsilon(\omega)} = n + \mathrm{i}\kappa$, where both n and κ are real-valued and positive. A linearly-polarized homogeneous plane-wave of frequency ω and vacuum wavelength $\lambda_0 = 2\pi c/\omega$ is normally incident from the air onto the coated metallic surface.

3 Pts a) Write expressions for the E- and H-fields in the medium of incidence (air), in the dielectric layer, and in the semi-infinite metallic substrate. Assume $\mu(\omega) = 1$ at the optical frequency ω, the speed of light in vacuum $c = 1/\sqrt{\mu_0\varepsilon_0}$, and the impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

3 Pts b) Match the boundary conditions at the top and bottom surfaces of the dielectric layer (located at $z = 0$ and $z = -d_0$) in order to arrive at relations among the various unknown parameters.

3 Pts c) Express the Fresnel reflection coefficient $\rho = E^{(\mathrm{r})}/E^{(\mathrm{i})}$ in terms of n, κ, n_0, d_0, ω, and c. The final result will be easier to analyze when expressed as a function of $\rho_1 = (1 - n_0)/(1 + n_0)$, $\rho_2 = [n_0 - (n + \mathrm{i}\kappa)]/[n_0 + (n + \mathrm{i}\kappa)]$, and the round-trip phase $\varphi_0 = 4\pi n_0 d_0/\lambda_0$.

1 Pts d) Assuming that the thickness d_0 of the dielectric layer is adjustable, when will the reflectance $R = |\rho|^2$ of the coated metallic surface reach a minimum or a maximum? What are the values of R_{min} and R_{max}?
