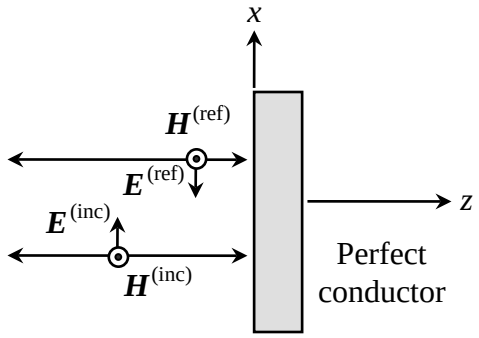
Spring 2015 Written Comprehensive Exam
Opti 501

System of units: MKSA

A monochromatic plane-wave of frequency ω traveling in free space is reflected at normal incidence from the flat surface of a perfect conductor. Denoting the speed of light in vacuum by $c = 1/\sqrt{\mu_0 \varepsilon_0}$ and the impedance of free space by $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, the incident and reflected E- and H-fields are given by

$$\begin{cases} \boldsymbol{E}^{(\mathrm{inc})}(\boldsymbol{r},t) = E_0 \hat{\boldsymbol{x}} \cos[(\omega/c)\, z - \omega t], \\ \boldsymbol{H}^{(\mathrm{inc})}(\boldsymbol{r},t) = (E_0/Z_0)\hat{\boldsymbol{y}} \cos[(\omega/c)\, z - \omega t]. \end{cases}$$

$$\begin{cases} \boldsymbol{E}^{(\mathrm{ref})}(\boldsymbol{r},t) = -E_0 \hat{\boldsymbol{x}} \cos[(\omega/c)\, z + \omega t], \\ \boldsymbol{H}^{(\mathrm{ref})}(\boldsymbol{r},t) = (E_0/Z_0)\hat{\boldsymbol{y}} \cos[(\omega/c)\, z + \omega t]. \end{cases}$$



2 Pts a) Write expressions for the total E-field and total H-field amplitudes in the half-space $z \le 0$.

Hint: $\cos a + \cos b = 2\cos[(a+b)/2]\cos[(a-b)/2]$; $\cos a - \cos b = -2\sin[(a+b)/2]\sin[(a-b)/2]$.

2 Pts b) Identify locations along the z-axis where the E-field is exactly equal to zero, and also locations where the H-field is exactly zero.

3 Pts c) Determine the local energy densities of the E- and H-fields in the half-space $z \le 0$.

3 Pts d) Find the total Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ in the half-space $z \le 0$, and explain the behavior of the electromagnetic energy as a function of time by analyzing the time-dependence of the Poynting vector in relation to the local energy densities of the E- and H-fields.

Hint: $2\sin(a)\cos(a) = \sin(2a)$.

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Inside a homogeneous, isotropic, non-magnetic, dielectric medium of refractive index $n(\omega)$, a monochromatic, homogeneous plane-wave propagates along the z-axis. The plane-wave is linearly-polarized along the x-axis, and the medium is transparent, that is, $n(\omega)$ is real and positive.

4 Pts a) Write expressions for the plane-wave's electric and magnetic fields, $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$, in terms of the E-field amplitude E_0, the angular frequency ω, the refractive index $n(\omega)$, the speed of light in vacuum c, and the impedance of free space Z_0.

2 Pts b) Express the dielectric function $\varepsilon(\omega)$ and the electric susceptibility $\chi(\omega)$ as functions of the refractive index $n(\omega)$.

4 Pts c) Write an expression for the polarization distribution $\boldsymbol{P}(\boldsymbol{r},t)$ in terms of E_0, ω, c, ε_0 and $n(\omega)$. What are the distributions of the electric bound-charge and bound-current densities, $\rho_{\text{bound}}(\boldsymbol{r},t)$ and $\boldsymbol{J}_{\text{bound}}(\boldsymbol{r},t)$, in the medium?
