Spring 2014 Written Comprehensive Exam
Opti 501

<u>System of units: MKSA</u>

All fields and all parameters in this problem are real-valued; do not use the complex notation.

The E-field of a monochromatic plane-wave propagating in free-space is given by

$$\boldsymbol{E}(\boldsymbol{r},t) = E_{x0}\cos(k_0 z - \omega t + \varphi_{x0})\,\hat{\boldsymbol{x}} + E_{y0}\cos\big(k_0 z - \omega t + \varphi_{y0}\big)\,\hat{\boldsymbol{y}}. \qquad (1)$$

1 Pt a) Identify the k-vector (both direction and magnitude), and relate it to the parameters of Eq. (1).

2 Pts b) Describe the state of polarization of the beam in terms of the values of $E_{x0}, \varphi_{x0}, E_{y0}, \varphi_{y0}$. (For example, describe the circumstances under which the beam would be linearly-polarized, or circularly-polarized, etc.)

2 Pts c) Use one of Maxwell's equations to determine the H-field $\boldsymbol{H}(\boldsymbol{r},t)$ of the plane-wave in terms of the parameters of Eq. (1).

2 Pts d) Write the complete expression of the plane-wave's Poynting vector, $\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)$, then explain its meaning and significance.

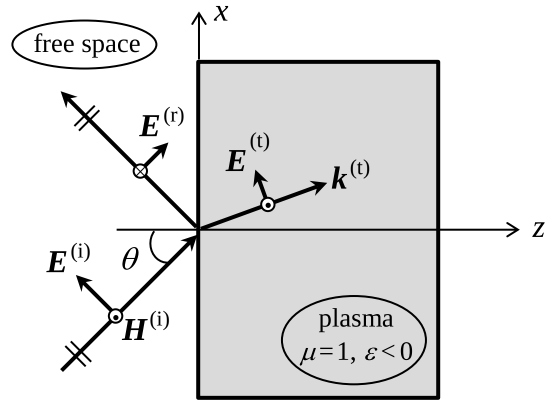
1 Pt e) Show that, for a circularly-polarized beam, the Poynting vector derived in part (d) is a constant, that is, it does *not* depend on z and t.

2 Pts f) Show that, for a linearly-polarized beam, the Poynting vector is a function of z and t. Thus, at a fixed instant of time, say, $t = t_0$, the Poynting vector $\boldsymbol{S}$ will have different values at different locations along the z-axis. Does this variation of $\boldsymbol{S}$ with z violate the law of conservation of energy? Explain.

Spring 2014 Written Comprehensive Exam
Opti 501

System of units: MKSA

A *p*-polarized monochromatic plane-wave arrives from free-space at the flat surface of a plasma at an oblique angle θ, as shown. The optical properties of the plasma are specified by its permittivity $\varepsilon(\omega)$, a real-valued *negative* entity, and by its permeability $\mu(\omega) = 1$.



2 Pts a) Write expressions for the E and H fields of the incident beam as functions of space and time.

2 Pts b) Write expressions for the E and H fields of the reflected beam as functions of space and time.

2 Pts c) Write expressions for the E and H fields of the beam transmitted into the plasma as functions of space and time. Identify the real and imaginary components of the k-vector, and relate them to the various parameters of the system.

2 Pts d) Match the boundary conditions at the plasma surface, and obtain expressions for the Fresnel reflection and transmission coefficients ρ_p and τ_p, respectively.

2 Pts e) Show that the reflectivity of the plasma is always 100%, irrespective of the incidence angle θ, or of the exact value of $\varepsilon(\omega)$. Explain the apparent contradiction between a 100% reflectance at the surface and the existence of electromagnetic field energy inside the plasma.