Spring 2012 Written Comprehensive Exam

Opti 501

<u>System of units: MKSA</u>

3 pts a) Write Maxwell's *macroscopic* equations in their most complete form, including contributions from free-charge and free-current densities, as well as those from polarization and magnetization. Explain the meaning of each symbol that appears in these equations.

2 pts b) Derive the charge-current continuity equation directly from Maxwell's equations, and explain the meaning of this equation (be brief yet precise).

3 pts c) Define the bound-electric-charge and bound-electric-current densities. Use these entities to eliminate the ***D*** and ***H*** fields from Maxwell's equations. (In other words, rewrite Maxwell's equations with the help of bound-charge and bound-current densities in such a way that only the ***E*** and ***B*** fields would appear in the equations.)

2 pts d) Show that the bound-charge and bound-current densities of part (c) satisfy their own charge-current continuity equation.

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Consider a homogeneous, linear, isotropic medium of permeability $\mu(\omega)=1.0$, real-valued and positive permittivity $\varepsilon(\omega)$, and refractive index $n(\omega)=\sqrt{\varepsilon(\omega)}$. Within this medium, two plane-waves having equal E-field amplitudes but differing frequencies propagate along the z-axis. Both plane-waves are linearly-polarized along the x-axis, their E-field amplitudes being E_o, and their respective frequencies being ω_1 and ω_2. The center frequency is $\omega_o=½(\omega_1+\omega_2)$, and the frequency difference $\Delta\omega=(\omega_2-\omega_1)$ is much smaller than ω_o.

3 pts a) Write expressions for the *real-valued* E- and H-field distributions of both plane-waves as functions of the space-time coordinates (x,y,z,t).

4 pts b) Write a complete expression for the rate of flow of electromagnetic energy (per unit area per unit time) associated with the superposition of the two plane-waves. Simplify the expression so that the energy flux associated with the beat-signal of frequency $\Delta\omega$ can be clearly identified. **Hint**: $\cos a \cos b = ½[\cos(a+b)+\cos(a-b)]$.

3 pts c) Ignoring the rapidly-oscillating terms in the expression obtained in part (b), show that the energy flow-rate associated with the beat signal moves along the z-axis at the group velocity V_g, which is derived from $n(\omega)$ in the vicinity of the beat signal's center frequency ω_o.
