Problem 1. Opti 501 Prelim, Spring 2011

System of units: MKSA

In this problem you are asked to describe and explain Maxwell's macroscopic equations in their most general differential form, and also to describe the related boundary conditions. Explain (in words) the meaning of the various parameters, operators, fields, and sources that appear in these equations. **Be brief but precise.**

(3 pts) a) Explain the meaning of the various terms in Maxwell's 1st equation, $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t)=\rho_{\text{free}}(\boldsymbol{r},t)$. How is the displacement field related to the permittivity of free space, ε_{o}, to the E-field, and to the polarization $\boldsymbol{P}(\boldsymbol{r},t)$? What boundary condition is derived from Maxwell's 1st equation? What does this boundary condition describe?

(4 pts) b) Explain the various terms in Maxwell's 2nd equation, $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)+\partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$. What boundary condition is derived from this equation, and what does the boundary condition imply for the fields and sources at and around the boundary?

(2 pts) c) Explain the various terms in Maxwell's 3rd equation, $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$. How is the B-field related to the permeability of free space, μ_{o}, to the H-field, and to the magnetization $\boldsymbol{M}(\boldsymbol{r},t)$? What boundary condition is derived from Maxwell's 3rd equation, and what does the boundary condition imply for the fields in the immediate vicinity of the boundary?

(1 pt) d) Explain the various terms in Maxwell's 4th equation, $\boldsymbol{\nabla}\cdot\boldsymbol{B}(\boldsymbol{r},t)=0$. What boundary condition is derived from this equation, and what does this boundary condition imply for the fields in the immediate vicinity of the boundary?

Problem 2. Opti 501 Prelim, Spring 2011

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An electromagnetic plane-wave of frequency ω resides in free space. There are no free charges and free currents, nor are there any polarization or magnetization in this space, i.e., $\rho_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{P}(\boldsymbol{r},t)=0$, and $\boldsymbol{M}(\boldsymbol{r},t)=0$. In general the plane-wave is elliptically polarized.

(2 pts) a) Write an expression for the *E*-field distribution in space and time. Identify the propagation direction (in terms of the *k*-vector), and the field amplitude. Specify the units of *k*, ω, and the *E*-field.

(1 pt) b) What property of the *E*-field distribution in part (a) distinguishes a linearly-polarized plane-wave from one that is circularly or elliptically polarized?

(1 pt) c) Write an expression for the *H*-field distribution in space and time. Specify the units of the *H*-field.

(1 pt) d) What constraint does Maxwell's 1st equation, $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t)=0$, impose on the various parameters of this plane-wave?

(1 pt) e) What constraint does Maxwell's 2nd equation, $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$, impose on the various parameters of this plane-wave?

(1 pt) f) What constraint does Maxwell's 3rd equation, $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$, impose on the various parameters of this plane-wave?

(1 pt) g) What constraint does Maxwell's 4th equation, $\boldsymbol{\nabla}\cdot\boldsymbol{B}(\boldsymbol{r},t)=0$, impose on the various parameters of this plane-wave?

(2 pts) h) Combining the various constraints obtained in parts (d)-(g) above, find the *dispersion relation* in vacuum, which is the relationship among $\boldsymbol{k}$, ω, and the vacuum speed of light, *c*.
