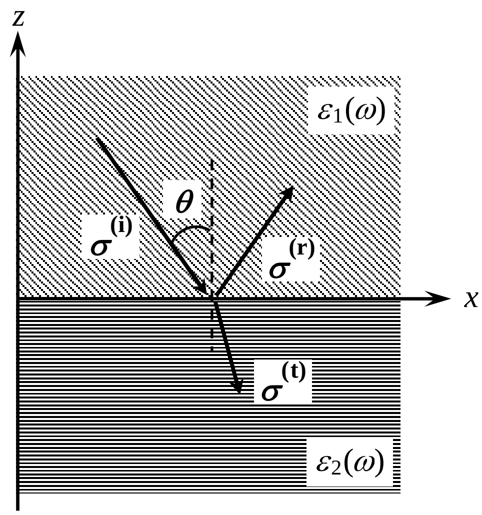
Opti 501 Prelims Questions (Spring 2010)

1) At the boundary between a transparent dielectric medium of refractive index $n_1=\sqrt{\varepsilon_1}$ and a second medium of *complex* refractive index $n_2+\mathrm{i}\kappa_2=\sqrt{\varepsilon_2}=\sqrt{\varepsilon'_2+\mathrm{i}\varepsilon''_2}$, the Fresnel reflection coefficients for *p*- and *s*-polarized plane-waves arriving at an incidence angle θ are given by:



$$\rho_\mathrm{p}=E^\mathrm{r}_{xo}/E^\mathrm{i}_{xo}=\frac{n_1\sqrt{\varepsilon_2-n_1^2\sin^2\theta}-\varepsilon_2\cos\theta}{n_1\sqrt{\varepsilon_2-n_1^2\sin^2\theta}+\varepsilon_2\cos\theta},$$

$$\rho_\mathrm{s}=E^\mathrm{r}_{yo}/E^\mathrm{i}_{yo}=\frac{n_1\cos\theta-\sqrt{\varepsilon_2-n_1^2\sin^2\theta}}{n_1\cos\theta+\sqrt{\varepsilon_2-n_1^2\sin^2\theta}}.$$

The relative permeabilities of both media are assumed to be unity, that is, $\mu_1=\mu_2=1$. Note that the dielectric constant ε_1 of the incidence medium is real-valued and positive, whereas that of the second medium, ε_2, is allowed to be complex-valued as well.

1 Pt a) What are the Fresnel reflection coefficients at normal incidence? If you found that $\rho_\mathrm{p}=\rho_\mathrm{s}$, explain why this result should be expected at normal incidence.

3 Pts b) Under what circumstances does one observe total internal reflection (TIR) in this problem? What is the critical incidence angle θ_c at which TIR begins? Do you expect the same critical angle for both p- and s-polarized plane-waves? Explain.

3 Pts c) Under what conditions does one arrive at Brewster's angle in this problem? Is there a Brewster's angle for s-polarized light? Why not?

1 Pt d) Can there be a Brewster's angle if the second medium is not transparent, that is, if $\kappa_2\neq 0$?

1 Pt e) What are the limiting values of ρ_p and ρ_s as one approaches grazing incidence (i.e., when $\theta\rightarrow 90°$)?

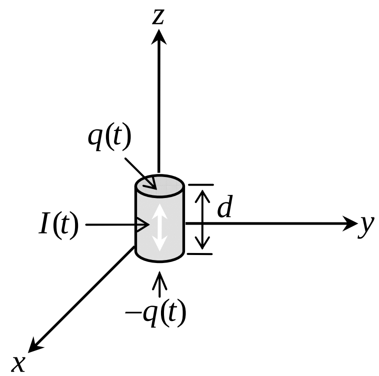
1 Pt f) Can there be total reflection (i.e., 100% reflectivity) at the interface if the second medium is not transparent, that is, if $\kappa_2\neq 0$?

2) A thin, short wire of length d, placed at the origin of a spherical coordinate system, carries the sinusoidal current $I(t) = I_o \sin(2\pi f t)$ along the z-axis. The scalar and vector potentials in the surrounding free-space are given by

$$\psi(\boldsymbol{r},t) = \frac{Z_o I_o d}{4\pi} \cos\theta \left\{ \frac{1}{r} \sin[2\pi f(t - \tfrac{r}{c})] - \left(\frac{\lambda_o}{2\pi r^2}\right) \cos[2\pi f(t - \tfrac{r}{c})] \right\},$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_o I_o d}{4\pi r} \sin[2\pi f(t - \tfrac{r}{c})] \hat{\boldsymbol{z}}.$$

In the above equations, $c = 1/\sqrt{\mu_o \varepsilon_o}$ is the speed of light in vacuum, while $Z_o = \sqrt{\mu_o/\varepsilon_o}$ is the impedance of the free space. The vacuum wavelength $\lambda_o = c/f$, and, in case you would like to work in spherical coordinates, $\hat{\boldsymbol{z}} = \cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$.



1 Pt a) Find the electric charge $\pm q(t)$ at the top and bottom of the oscillator.

1 Pt b) What is the strength of this electric dipole oscillator, $\boldsymbol{p}(t)$?

3 Pts c) Find the magnetic field distribution $\boldsymbol{H}(\boldsymbol{r},t)$ in the entire space surrounding the oscillator.

3 Pts d) Find the electric field distribution $\boldsymbol{E}(\boldsymbol{r},t)$ in the entire space surrounding the oscillator.

2 Pts e) Determine the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ in the far-field region of the dipole oscillator.