Opti 501 Prelims, Spring 2009

1) A monochromatic plane-wave (angular frequency $= \omega$) propagates in a homogeneous, isotropic medium specified by its relative permittivity and permeability, $\varepsilon(\omega)$ and $\mu(\omega)$. The constitutive relations, therefore, are $\boldsymbol{D}(\boldsymbol{r},\omega)=\varepsilon_o\varepsilon(\omega)\boldsymbol{E}(\boldsymbol{r},\omega)$ and $\boldsymbol{B}(\boldsymbol{r},\omega)=\mu_o\mu(\omega)\boldsymbol{H}(\boldsymbol{r},\omega)$. In general, $\varepsilon(\omega)$ and $\mu(\omega)$ are complex functions of the frequency ω. There are no free charges nor free currents in the system, that is, $\rho_{\text{free}}=0$, $\boldsymbol{J}_{\text{free}}=0$.

(2 pts) a) Using the complex notation, write the most general expressions for the plane-wave's $E(\boldsymbol{r},t)$ and $H(\boldsymbol{r},t)$. The k-vector may be written as $k_o\boldsymbol{\sigma}$, where $k_o=\omega/c$ is the magnitude of the k-vector in vacuum, while $\boldsymbol{\sigma}=\sigma_x\hat{\boldsymbol{x}}+\sigma_y\hat{\boldsymbol{y}}+\sigma_z\hat{\boldsymbol{z}}$ is a complex vector in 3-dimensional space. Also, in general, the field amplitudes $\boldsymbol{E}_o=E_{xo}\hat{\boldsymbol{x}}+E_{yo}\hat{\boldsymbol{y}}+E_{zo}\hat{\boldsymbol{z}}$ and $\boldsymbol{H}_o=H_{xo}\hat{\boldsymbol{x}}+H_{yo}\hat{\boldsymbol{y}}+H_{zo}\hat{\boldsymbol{z}}$ are complex vectors in 3D space.

(4 pts) b) Use Maxwell's equations to relate the various parameters of the fields $(\boldsymbol{E}_o,\boldsymbol{H}_o,k_o,\boldsymbol{\sigma})$ to each other and to the parameters of the medium (ε,μ). Use the standard relations $c=1/\sqrt{\mu_o\varepsilon_o}$ and $Z_o=\sqrt{\mu_o/\varepsilon_o}$ to simplify the resulting expressions. Remember that $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})=(\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B}-(\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$.

(2 pts) c) Express the (complex) refractive index of the medium, $n(\omega)$, in terms of $\varepsilon(\omega)$ and $\mu(\omega)$.

(2 pts) d) Assuming $\sigma_x=\sigma_y=0$, determine the (complex) impedance of the medium, $Z(\omega)=E_x/H_y=-E_y/H_x$, in terms of $\varepsilon(\omega)$, $\mu(\omega)$, and Z_o.

Hint: In the MKSA system of units, Maxwell's macroscopic equations are written

$$\nabla\cdot\boldsymbol{D}=\rho_{\text{free}};\qquad \nabla\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}+\partial\boldsymbol{D}/\partial t;\qquad \nabla\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t;\qquad \nabla\cdot\boldsymbol{B}=0.$$

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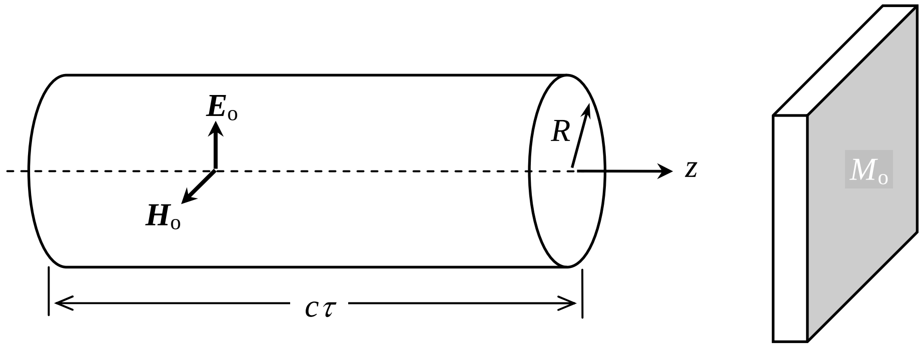
2) A pulse of light, having duration τ and a circular cross-section with radius R, propagates in the free space along the z-axis, as shown. The pulse is long and wide, so that diffraction effects can be ignored. (The beam may be treated effectively as a monochromatic plane-wave albeit one that has a finite duration and a finite cross-sectional area.) For simplicity, assume the beam is linearly polarized, with E- and H-field amplitudes specified as E_o (volt/meter) and H_o (ampere/meter). The impedance of the free-space is $Z_o \approx 377\,\Omega$.



(4 pts) a) Find the pulse's time-averaged Poynting vector $\langle \boldsymbol{S} \rangle$, the total energy content of the pulse, and its total momentum.

(3 pts) b) Let the pulse be reflected from a massive mirror (i.e., mass $M_o \to \infty$) whose reflectivity is 100%. What is the mechanical momentum acquired by the mirror after the entire pulse has been reflected? How much kinetic energy does the mirror acquire in this process? If the mirror happens to have a *finite* mass M_o, where does its kinetic energy come from?

(3 pts) c) Instead of a perfect reflector, assume now that the object of mass M_o is a perfect *absorber*. Once the pulse has been fully absorbed, what will be the mechanical momentum of the absorber? What will be its kinetic energy? Where does this kinetic energy come from? What happens to the remaining energy of the light pulse?