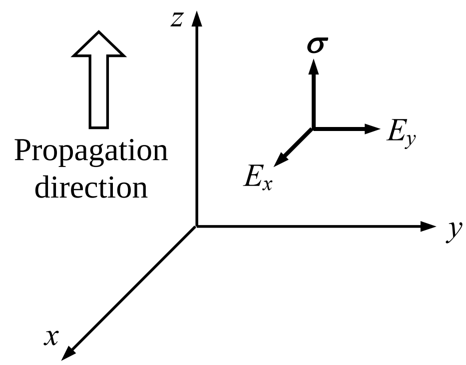
Opti 501 Prelims, Spring 2008



1) A homogeneous plane wave propagates in free space along the z-axis. The oscillation frequency is $\omega = 2\pi f$, the wavelength is $\lambda_o = c/f$, the propagation constant is $k_o = 2\pi/\lambda_o$, the speed of light is c, and the impedance of the free space is Z_o. The only restrictions on the fields are those imposed by Maxwell's equations.

(2 pts) a) Write expressions for the propagation vector $\boldsymbol{\sigma}$, the E-field amplitude $\boldsymbol{E}_o = E_{xo}\hat{\boldsymbol{x}} + E_{yo}\hat{\boldsymbol{y}} + E_{zo}\hat{\boldsymbol{z}}$, and the H-field amplitude $\boldsymbol{H}_o = H_{xo}\hat{\boldsymbol{x}} + H_{yo}\hat{\boldsymbol{y}} + H_{zo}\hat{\boldsymbol{z}}$, consistent with Maxwell's equations.

(2 pts) b) What conditions should E_{xo} and E_{yo} satisfy for the plane-wave to be linearly polarized?

(2 pts) c) What conditions should E_{xo} and E_{yo} satisfy for the plane-wave to be circularly polarized?

(2 pts) d) Let $E_{xo} = |E_{xo}|\exp(i\phi_{xo})$ and $E_{yo} = |E_{yo}|\exp(i\phi_{yo})$. Assuming $|E_{xo}| > |E_{yo}|$ and $\phi_{xo} - \phi_{yo} = 90°$, what is the polarization ellipticity η of the plane-wave?

(2 pts) e) Starting from the formula $<\boldsymbol{S}(\boldsymbol{r}, t)> = \frac{1}{2} Re(\boldsymbol{E} \times \boldsymbol{H}^*)$ and showing every step of the calculation, derive an expression for the time-averaged Poynting vector $<\boldsymbol{S}(\boldsymbol{r}, t)>$ in terms of E_{xo} and E_{yo}.

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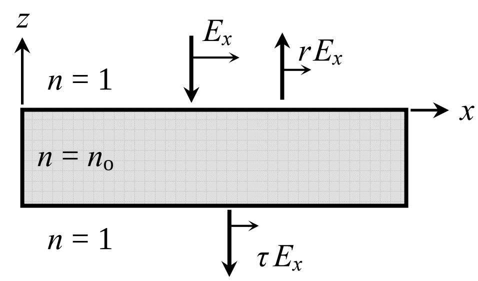

2) A monochromatic plane-wave is normally incident upon a transparent dielectric slab (i.e., real-valued refractive index n_o). The incident beam is linearly polarized, with E-field along the x-axis, as shown. The slab's reflection and transmission coefficients are r and τ, respectively.

(2.5 pts) a) Express the average rate of flow of optical energy $\langle S_z \rangle$ (i.e., energy per unit area per unit time) in the incident beam in terms of E_x.

(2.5 pts) b) Show that the fraction of reflected optical energy is $R = |r|^2$, while the fraction of transmitted optical energy is $T = |\tau|^2$.

(2.5 pts) c) Use the conservation of energy to derive a relationship between R and T.

(2.5 pts) d) Use the conservation of momentum to find the radiation pressure (i.e., time-averaged force per unit area) on the slab in terms of the incident beam's $\langle S_z \rangle$ and the slab's R and T.