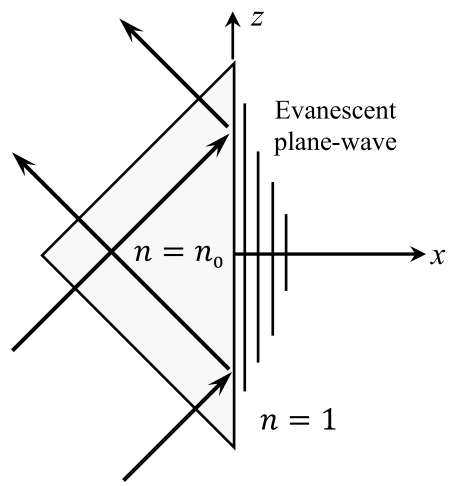
PhD Qualifying Exam, Fall 2021

Opti 501

System of units: SI (or MKSA)



1) A homogeneous plane-wave undergoes total internal reflection within a glass prism of refractive index $n = n_0$, as shown. As usual, it is assumed that $\mu(\omega) = 1.0$ at optical frequencies, and that $\boldsymbol{\sigma} = \boldsymbol{k}/k_0$ is the normalized k-vector. The resulting evanescent plane-wave in the free-space region beyond the prism (i.e., $x \geq 0$) has the following E- and H-fields:

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \exp[\mathrm{i}(k_0 \boldsymbol{\sigma} \cdot \boldsymbol{r} - \omega t)],$$

$$\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{H}_0 \exp[\mathrm{i}(k_0 \boldsymbol{\sigma} \cdot \boldsymbol{r} - \omega t)].$$

Let $\boldsymbol{\sigma} = \mathrm{i}\sigma_x \hat{\boldsymbol{x}} + \sigma_z \hat{\boldsymbol{z}}$, while $\boldsymbol{E}_0 = E_{x0}\hat{\boldsymbol{x}} + E_{y0}\hat{\boldsymbol{y}} + E_{z0}\hat{\boldsymbol{z}}$ and $\boldsymbol{H}_0 = H_{x0}\hat{\boldsymbol{x}} + H_{y0}\hat{\boldsymbol{y}} + H_{z0}\hat{\boldsymbol{z}}$. In general, σ_x and σ_z are real-valued, while $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ are complex.

a) What is the relationship between σ_x and σ_z?

b) What does Maxwell's first equation say about the relation between E_{x0} and E_{z0}?

c) What does Maxwell's 4$^{\text{th}}$ equation say about the relation between H_{x0} and H_{z0}?

d) What is the relation between (E_{x0}, E_{y0}, E_{z0}) and (H_{x0}, H_{y0}, H_{z0}) based on Maxwell's 3$^{\text{rd}}$ equation?

e) If $E_{z0} = 0$, which components of $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ will vanish as well?

f) If $H_{z0} = 0$, which components of $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ will vanish as well?

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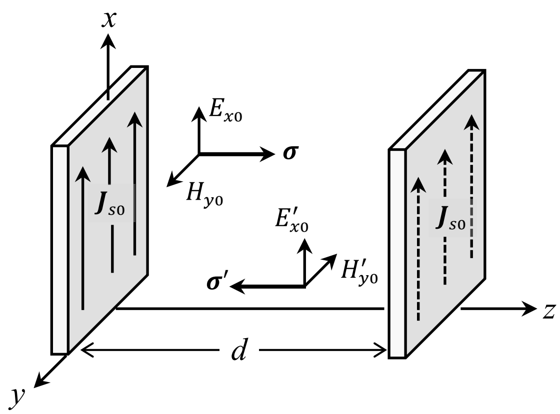
2) Two counter-propagating, linearly-polarized, homogeneous plane waves, both having frequency ω and wavelength λ_0, are trapped in the free-space region between a pair of perfectly conducting mirrors, as shown. The mirrors are parallel to the xy-plane, their separation d being an integer-multiple of $\lambda_0/2$. The propagation directions are $\boldsymbol{\sigma} = \hat{\boldsymbol{z}}$ and $\boldsymbol{\sigma}' = -\hat{\boldsymbol{z}}$. The (complex) E-field amplitudes $E_{x0} = |E_{x0}| \exp(\mathrm{i}\varphi_0)$ and $E'_{x0} = |E'_{x0}| \exp(\mathrm{i}\varphi'_0)$ have equal magnitudes; that is, $|E_{x0}| = |E'_{x0}|$.



a) Write expressions for the total E- and H-fields in the cavity between the mirrors. (These expressions can be further simplified by taking into account the conditions at the mirrors.)

b) In terms of $|E_{x0}|$, find the magnitude of the surface-current-density $J_{s0}\hat{\boldsymbol{x}}$ on the mirror surfaces.

c) Determine the total E- and H-field energies trapped within the cavity. Show that, at those instants of time when the E-field energy is zero, the H-field energy is at a maximum, and vice-versa.

d) Write an expression for the Poynting vector $\boldsymbol{S}(z,t)$ inside the cavity. Interpret the oscillations of $\boldsymbol{S}(z,t)$ at a fixed location in space as time varies during each oscillation period.

Hint: $\cos a + \cos b = 2\cos[½(a+b)]\cos[½(a-b)]$;

$\cos a - \cos b = -2\sin[½(a+b)]\sin[½(a-b)]$.
