Fall 2015 Written Comprehensive Exam
Opti 501

<u>System of units: MKSA</u>

2 Pts a) The charge-current continuity equation is written

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{r},t)+\partial\rho(\boldsymbol{r},t)/\partial t=0.$$

Explain in a few sentences the physical meaning of the equation and also the meaning of the various terms and symbols that appear in the equation. Specify the units of $\boldsymbol{J}$ and ρ.

3 Pts b) Derive the charge-current continuity equation for free charge-density $\rho_{\text{free}}(\boldsymbol{r},t)$ and free current-density $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)$ from Maxwell's *microscopic* equations.

Note: The microscopic equations of Maxwell do *not* contain polarization and magnetization.

2 Pts c) Write down the definitions of bound electric charge-density and bound electric current-density, then relate them to each other via the corresponding continuity equation.

3 Pts d) Derive the charge-current continuity equation for bound electric charge and bound electric current from Maxwell's *macroscopic* equations.

Note: You may now set $\rho_{\text{free}}=0$ and $\boldsymbol{J}_{\text{free}}=0$ in Maxwell's macroscopic equations.

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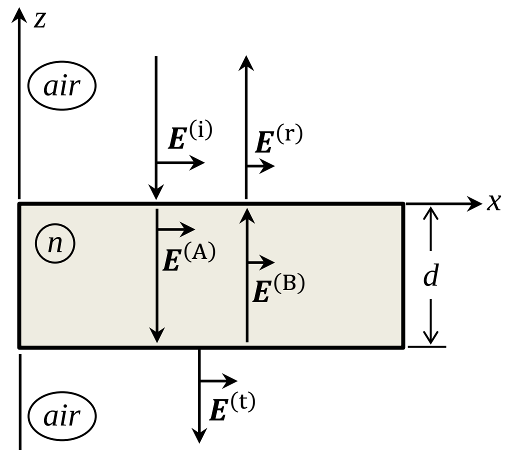

A monochromatic, linearly-polarized, homogeneous plane-wave is normally incident on a slab of transparent dielectric material of thickness d and (real-valued) refractive index n, as shown.

3 Pts a) Write expressions for the E- and H-fields in the medium of incidence (air), in the dielectric slab, and in the region below the slab, in which the transmitted beam emerges.

(You may assume $\mu = 1$, incident angular frequency $= \omega$, speed of light in vacuum $c = 1/\sqrt{\mu_0 \varepsilon_0}$, and impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.)

2 Pts b) Match the boundary conditions at the top surface $(z = 0)$ and at the bottom surface $(z = -d)$ of the slab in order to arrive at relations among the various unknown parameters.

2 Pts c) Find expressions for the Fresnel reflection and transmission coefficients $\rho = E^{(\mathrm{r})}/E^{(\mathrm{i})}$ and $\tau = E^{(\mathrm{t})}/E^{(\mathrm{i})}$ in terms of n, d, ω, and c. (**Note**: In air, the incident wavelength is $\lambda_0 = 2\pi c/\omega$.)

2 Pts d) Under what circumstances will the reflectance $R = |\rho|^2$ of the slab be at a minimum? What is the value of $R_{\min}$?

1 Pt e) When will the reflectance of the slab be at a maximum, and what is the value of $R_{\max}$?