Fall 2014 Written Comprehensive Exam
Opti 501

System of units: MKSA

In this problem you are asked to derive two different versions of the Poynting theorem, one with which you are familiar, and a second version which you may not have seen before. Both versions are derived directly from Maxwell's equations. You should use the general definitions for the sources $\rho_{\text{free}}(\boldsymbol{r},t)$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)$, $\boldsymbol{P}(\boldsymbol{r},t)$, and $\boldsymbol{M}(\boldsymbol{r},t)$; in other words, do ***not*** assume that the material media are linear, isotropic, homogeneous, etc.

2 Pts a) Write Maxwell's macroscopic equations in their most general form. Make sure to specify the relationship among the $\boldsymbol{D}, \boldsymbol{E}, \boldsymbol{P}$ fields, and also that among the $\boldsymbol{B}, \boldsymbol{H}, \boldsymbol{M}$ fields.

2 Pts b) Eliminate $\boldsymbol{P}(\boldsymbol{r},t)$ and $\boldsymbol{M}(\boldsymbol{r},t)$ from Maxwell's equations by introducing the notions of bound *electric* charge and bound *electric* current. (At this point the equations should no longer contain the $\boldsymbol{D}$ and $\boldsymbol{H}$ fields.)

2 Pts c) Dot-multiply one of the curl equations into $\boldsymbol{E}$ and the other curl equation into $\boldsymbol{B}$, then subtract one equation from the other. Manipulate the resulting equation algebraically to produce one version of the Poynting theorem.

2 Pts d) Explain (in words) the meaning of the various terms in the Poynting theorem obtained in (c).

2 Pts e) Repeat parts (b), (c), and (d), this time introducing the notions of bound *electric* charge and bound *electric* current to replace $\boldsymbol{P}$, and also bound *magnetic* charge and bound *magnetic* current to replace $\boldsymbol{M}$ in Maxwell's equations. (At this point the equations should no longer contain the $\boldsymbol{D}$ and $\boldsymbol{B}$ fields.) Dot-multiplication should be carried out with the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields.

Hint: The following vector identity will be useful: $\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) = \boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B})$.

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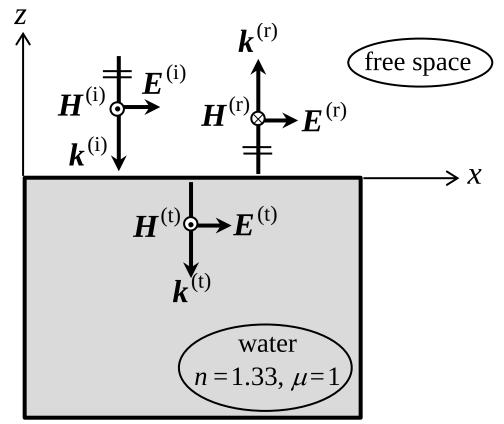
A monochromatic, linearly-polarized, homogeneous plane-wave is normally incident from free-space onto the flat surface of pure water (refractive index $n = 1.33$, relative permeability $\mu = 1$).



2 Pts a) Write expressions for the incident, reflected, and transmitted plane-waves. Specify the k-vector of each beam in terms of the frequency ω, the speed of light in vacuum, c, and the corresponding refractive index.

3 Pts b) Use the Fresnel reflection and transmission coefficients to relate the amplitudes of the E- and H-fields of the reflected and transmitted beams to those of the incident beam.

3 Pts c) Relate the time-averaged rate-of-flow of electromagnetic energy in each of the three beams to the corresponding E-field intensity $|\boldsymbol{E}|^2$.

2 Pts d) Confirm the conservation of energy by showing that the sum of the rates of reflected and transmitted energy flow is precisely equal to the rate of incident energy flow.