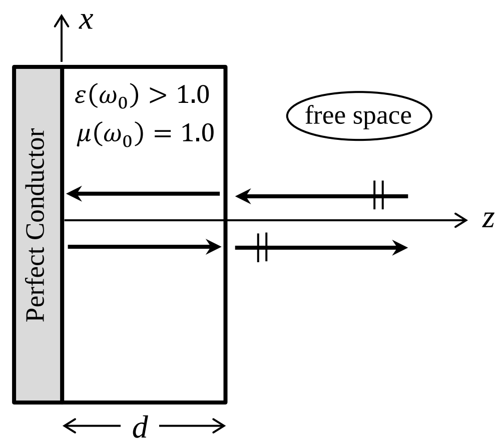
Fall 2013 Written Comprehensive Exam
Opti 501

System of units: MKSA

A homogeneous, monochromatic plane-wave arrives at normal incidence on a perfect electrical conductor coated with a dielectric layer of thickness d, as shown. The boundary between the dielectric and the perfect conductor is the xy-plane at $z = 0$. The refractive index of the dielectric is given by $n(\omega) = \sqrt{\mu(\omega)\varepsilon(\omega)}$, where the relative permittivity $\varepsilon(\omega)$ is real-valued and greater than 1.0, while the relative permeability $\mu(\omega)$ may, as usual, be set equal to 1.0 at optical frequencies. Inside the dielectric layer, the (real-valued) E-field amplitude is given by $\boldsymbol{E}(\boldsymbol{r},t) = E_1\hat{\boldsymbol{x}}\sin(k_1 z + \varphi_1)\cos(\omega_0 t)$.



1 Pt a) Use the relevant boundary condition at the surface of the perfect conductor to determine the value of φ_1.

2 Pts b) Use Maxwell's equation $\boldsymbol{\nabla} \times \boldsymbol{E}(\boldsymbol{r},t) = -\partial \boldsymbol{B}(\boldsymbol{r},t)/\partial t$ to determine the H-field amplitude $\boldsymbol{H}(\boldsymbol{r},t)$ inside the dielectric.

2 Pts c) Use Maxwell's equation $\boldsymbol{\nabla} \times \boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) + \partial \boldsymbol{D}(\boldsymbol{r},t)/\partial t$ in conjunction with the results obtained in parts (a) and (b) to determine the propagation constant k_1 in terms of ω_0, $n(\omega_0)$, and c, the speed of light in vacuum.

1 Pt d) Show that the remaining Maxwell equations, $\boldsymbol{\nabla} \cdot \boldsymbol{D}(\boldsymbol{r},t) = \rho_{\text{free}}(\boldsymbol{r},t)$ and $\boldsymbol{\nabla} \cdot \boldsymbol{B}(\boldsymbol{r},t) = 0$, are automatically satisfied.

In the free-space region $z > d$, The E-field amplitude is $\boldsymbol{E}(\boldsymbol{r},t) = E_0\hat{\boldsymbol{x}}\sin(k_0 z + \varphi_0)\cos(\omega_0 t)$. (This standing wave is the result of interference between the incident and reflected plane-waves.)

2 Pts e) Use Maxwell's equations as before to determine $\boldsymbol{H}(\boldsymbol{r},t)$ in the free-space region, and also to relate the propagation constant k_0 to ω_0 and c.

2 Pts f) Invoke the relevant boundary conditions at $z = d$, the interface between free-space and the dielectric layer, to relate E_1/E_0 and φ_0 to the various parameters of the system.

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}.$$

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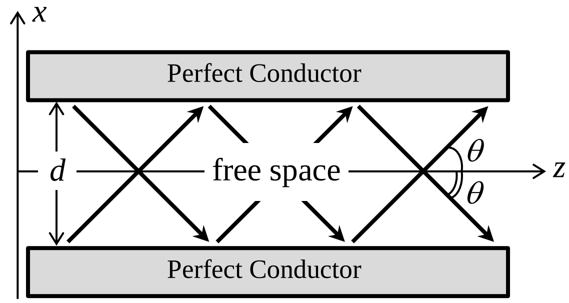

In the free-space region between two perfectly electrically conducting parallel plates, a guided electromagnetic wave propagates along the z-axis, as shown. The plates are separated by a distance d along the x-axis, and the guided wave is single-mode, monochromatic, and p-polarized. The (real-valued) magnetic field of the guided mode is $\boldsymbol{H}(\boldsymbol{r},t) = H_0\hat{\boldsymbol{y}}\cos(k_x x)\sin(k_z z - \omega_0 t)$.

2 Pts a) Use Maxwell's equation $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) + \partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$ to determine the E-field profile of the guided mode.

2 Pts b) Use Maxwell's equation $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$ to determine the relationship among k_x, k_z, ω_0, and c, the speed of light in vacuum.

2 Pts c) What values of k_x are admissible if the guided mode is to satisfy the boundary conditions at the inner surfaces of the perfect conductors?

3 Pts d) Find the surface charge and current densities $\sigma_s(x = \pm\frac{1}{2}d, y, z, t)$ and $\boldsymbol{J}_s(x = \pm\frac{1}{2}d, y, z, t)$ at the inner surfaces of the perfect conductors.

1 Pt e) Show that σ_s and $\boldsymbol{J}_s$ obtained in part (d) satisfy the charge-current continuity equation.

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

$$\boldsymbol{\nabla}\times\boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}.$$
