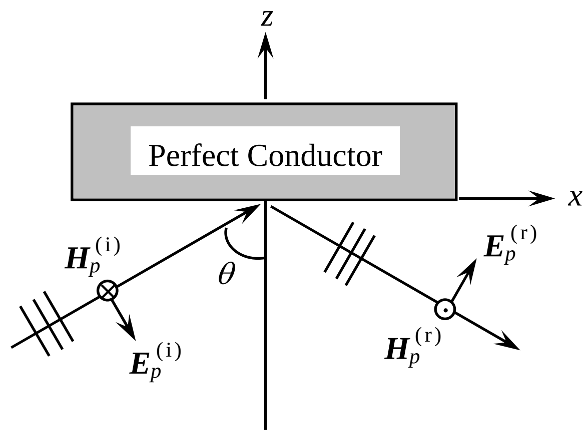
Problem 1. Opti 501 Prelim, Fall 2010

System of units: MKSA

A monochromatic plane-wave of frequency ω arrives from free space at an angle of incidence θ at the flat surface of a perfect conductor. The plane-wave is linearly polarized in the *p*-direction, as shown in the figure.



(1 pt) a) Write the expression for the incident plane-wave, identifying its *k*-vector, its *E*-field, and its *H*-field. (The *E*-field is assumed to be known, but the *H*-field must be related to the *E*-field.)

(2 pts) b) Write the expression for the reflected plane-wave, identifying its *k*-vector, its *E*-field, and its *H*-field. (The reflected *E*-field must be related to the incident *E*-field via the boundary conditions at the surface of the perfect conductor.)

(2 pts) c) Verify conservation of energy by checking that the incident and reflected beams have the same (time-averaged) rate of flow of energy per unit cross-sectional area per unit time.

(2 pts) d) Find the surface-current density $\boldsymbol{J}_s(x,y,t)$ at the surface of the perfect conductor.

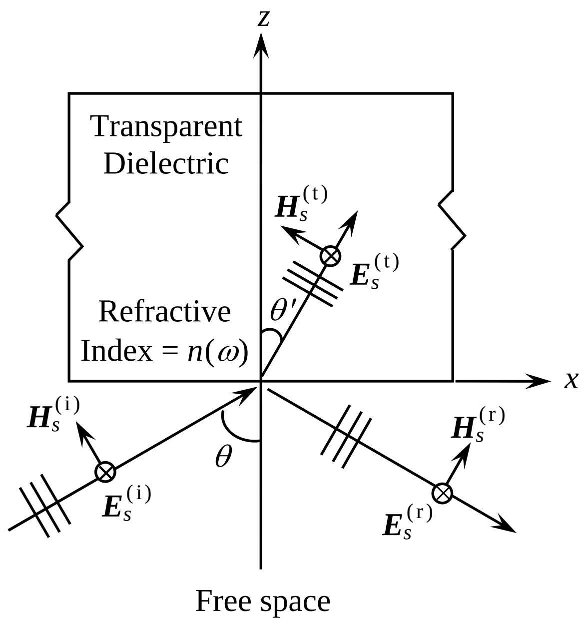
(2 pts) e) Find the surface-charge density $\sigma_s(x,y,t)$ at the surface of the perfect conductor.

(1 pt) f) Verify that the surface charge- and current-densities obtained in parts (d) and (e) above satisfy the charge-current continuity equation, namely, $\boldsymbol{\nabla}\cdot\boldsymbol{J}_s(x,y,t)+\partial\sigma_s(x,y,t)/\partial t=0$.

Problem 2. Opti 501 Prelim, Fall 2010

System of units: MKSA

A monochromatic plane-wave of frequency ω arrives from free space at an angle of incidence θ at the flat surface of a transparent, semi-infinite, dielectric medium of refractive-index $n(\omega)=\sqrt{\varepsilon(\omega)}$. The plane-wave is linearly polarized in the *s*-direction, as shown in the figure, and the magnetic permeability of the dielectric is assumed to be unity, i.e., $\mu(\omega)=1$.



(4 pt) a) Write expressions for the incident, reflected, and transmitted plane-waves, identifying their respective *k*-vectors, *E*-fields, and *H*-fields. (The incident *E*-field is assumed to be known, but all the other *E*- and *H*-fields must eventually be related to the known parameters.)

(4 pts) b) Write the relevant boundary conditions and determine the Fresnel reflection and transmission coefficients, ρ_s and τ_s, for the *s*-polarized incident plane-wave.

(2 pts) c) Verify conservation of energy by calculating the rate-of-flow of energy per unit cross-sectional area per unit time for the incident, reflected, and transmitted beams.