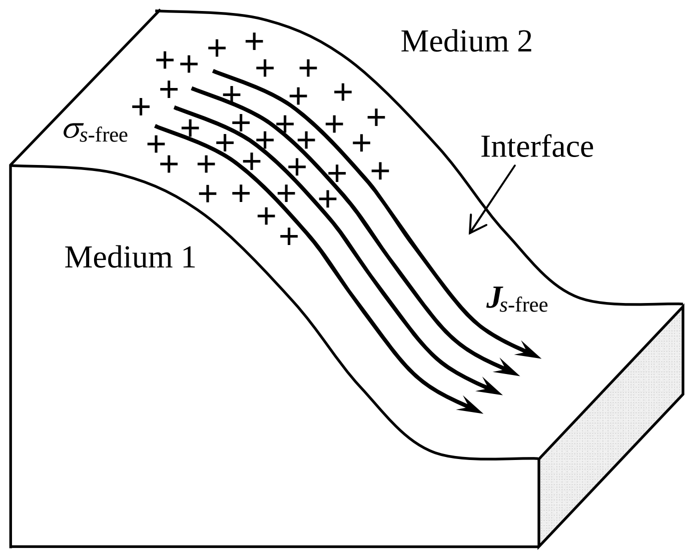
**Problem 1. Opti 501 Prelims, Fall 2009**

**System of units: MKSA**

Corresponding to the four Maxwell equations are four boundary conditions that relate the field components $\boldsymbol{E}_{||}$, $\boldsymbol{H}_{||}$, $\boldsymbol{D}_{\perp}$ and $\boldsymbol{B}_{\perp}$ on the two sides of a sharply defined interface between two neighboring media. The subscripts || and $\perp$ identify the local field components parallel and perpendicular to the interface, respectively. In general, the interface may contain a surface-charge-density $\sigma_{s\text{-free}}(\boldsymbol{r},t)$, and be host to a surface-current-density $\boldsymbol{J}_{s\text{-free}}(\boldsymbol{r},t)$. Here $\boldsymbol{r}=(x,y,z)$ is an arbitrary point at the interface, and $t$ is an arbitrary instant of time. In what follows, $\boldsymbol{r}^+$ will be a point immediately above the interface, while $\boldsymbol{r}^-$ will be the corresponding point immediately below the interface.




(2 pts) a) Use Maxwell's first equation, $\nabla\cdot\boldsymbol{D}=\rho_{\text{free}}$, to relate $\boldsymbol{D}_{\perp}(\boldsymbol{r}^+,t)$ and $\boldsymbol{D}_{\perp}(\boldsymbol{r}^-,t)$.

(4 pts) b) Use Maxwell's second equation, $\nabla\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}+\partial\boldsymbol{D}/\partial t$, to relate $\boldsymbol{H}_{||}(\boldsymbol{r}^+,t)$ and $\boldsymbol{H}_{||}(\boldsymbol{r}^-,t)$.

(2 pts) c) Use Maxwell's third equation, $\nabla\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t$, to relate $\boldsymbol{E}_{||}(\boldsymbol{r}^+,t)$ and $\boldsymbol{E}_{||}(\boldsymbol{r}^-,t)$.

(2 pts) d) Use Maxwell's fourth equation, $\nabla\cdot\boldsymbol{B}=0$, to relate $\boldsymbol{B}_{\perp}(\boldsymbol{r}^+,t)$ and $\boldsymbol{B}_{\perp}(\boldsymbol{r}^-,t)$.

**Problem 2. Opti 501 Prelims, Fall 2009**

**System of units: MKSA**

A linearly-polarized, monochromatic plane-wave propagates along the $x$-axis, its $E$-field amplitude being $\boldsymbol{E}(x,t)=E_o\cos\{\omega[t-n(\omega)x/c]\}\hat{\boldsymbol{y}}$. The host medium is a homogeneous, isotropic, non-magnetic (i.e., $\mu=\mu_o$), transparent dielectric, whose frequency-dependent refractive index is specified as $n(\omega)=\sqrt{\varepsilon(\omega)}$.

(3 pts) a) Find the magnetic field $\boldsymbol{H}(x,t)$ of the plane-wave in terms of $E_o$, $c$, $\omega$, $n(\omega)$, and the impedance of the free space $Z_o=\sqrt{\mu_o/\varepsilon_o}$.

(3 pts) b) Find the Poynting vector $\boldsymbol{S}(x,t)$ of the above plane-wave, then determine the time-averaged rate-of-flow of optical energy (per unit area per unit time) along the $x$-axis.

(4 pts) c) Assume a second plane-wave, *identical* with the one above *except* for its frequency $\omega'$ differing slightly from $\omega$, is co-propagating with the above plane-wave. Write an expression for the combined $E$-field of the superposed plane-waves. From this expression, identify the carrier and the envelope of the beat waveform. In terms of $c$, $\omega_c=½(\omega+\omega')$, $\Delta\omega=\omega'-\omega$, $n(\omega_c)$ and $\mathrm{d}n(\omega)/\mathrm{d}\omega$, what is the *phase* and *group* velocity of the combined waveform?