**Opti 501, Fall 2007**

System of Units: MKSA

Problem 1.

15% a) Describe in words the meaning of Maxwell's first equation, $\nabla \cdot \boldsymbol{D} = \rho$. What is $\rho$, what is $\nabla \cdot$, what is $\boldsymbol{D}$, and how is $\boldsymbol{D}$ related to the electric field $\boldsymbol{E}$?

20% b) Describe in words the meaning of Maxwell's second equation, $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D}/\partial t$. What is $\boldsymbol{H}$, what is $\nabla \times$, what is $\boldsymbol{J}$, and how is $\boldsymbol{J}$ related to charge density $\rho$ and the velocity $\boldsymbol{V}$ of these charges?

20% c) Starting with Maxwell's first and second equations, derive the "charge continuity" equation.

15% d) Describe in words the meaning of Maxwell's third equation, $\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$. What is $\boldsymbol{B}$, and how is it related to $\boldsymbol{H}$? Under what circumstances can one define a scalar potential $\psi$ such that the electric field may be expressed as $\boldsymbol{E} = -\nabla \psi$?

15% e) Describe in words the meaning of Maxwell's fourth equation, $\nabla \cdot \boldsymbol{B} = 0$.

15% f) Are Maxwell's third and fourth equations consistent with each other in the same sense that the first and second equations were shown to be consistent in part (c) above? Is the fourth equation implicit in the third equation?

**Hint**: For an arbitrary vector field $\boldsymbol{A}(\boldsymbol{r}, t)$, application of Stokes' theorem to a closed surface yields $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$.

**Opti 501, Fall 2007**

System of Units: MKSA

Problem 2.

A monochromatic plane-wave (wavelength = $\lambda$) is incident at an angle $\theta$ on a semi-infinite dielectric medium of refractive index $n$, as shown in the figure. The medium of incidence is free-space, the refractive index $n$ is real-valued, the plane of incidence is $xz$, and the incident beam's state of polarization is linear, with the $E$-field confined to the plane of incidence (i.e., p-polarized). In the figure, the various $E$ and $H$-fields are identified with subscripts $i$ (incident), $r$ (reflected), and $t$ (transmitted). The transmitted beam's angle with the surface normal is $\theta'$.

40% a) Write the complex waveforms of the $E$- and $H$-fields for the incident, reflected, and transmitted beams. (Use $\theta$, $\theta'$, $n$, the speed of light $c = 1/\sqrt{\mu_o \varepsilon_o}$, and the impedance of the free-space $Z_o = \sqrt{\mu_o/\varepsilon_o}$ to minimize the number of independent variables at this stage. $Z_o$ is the ratio $E/H$ for a plane-wave in the free space.)

40% b) Use the relevant boundary conditions at the entrance surface to determine the relation between $\theta$ and $\theta'$ (i.e., Snell's law), and also to relate the various components of the $E$- and $H$-fields to each other.

20% c) Solving the set of equations obtained in (b), determine the Fresnel reflection and transmission coefficients at the dielectric surface.