Opti 501 **Midterm Exam** (4/20/2010) **Time: 2hours**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1. Start with the complete set of Maxwell's macroscopic equations for a system containing the sources $\rho_{\text{free}}(\boldsymbol{r},t)$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)$, $\boldsymbol{P}(\boldsymbol{r},t)$, and $\boldsymbol{M}(\boldsymbol{r},t)$.

3 Pts a) Eliminate the fields $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$ from all four equations, using an appropriate set of bound charge and bound current densities.

3 Pts b) Transform the equations obtained in part (a) to the Fourier domain.

4 Pts c) Without introducing any potentials, find the solutions to the equations obtained in part (b), that is, find expressions for $\boldsymbol{D}(\boldsymbol{k},\omega)$ and $\boldsymbol{H}(\boldsymbol{k},\omega)$ in terms of the known source distributions.

Hint: The vector identity $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$ may be helpful in answering part (c).

Problem 2. A point-charge q moves along the x-axis with a constant velocity V.

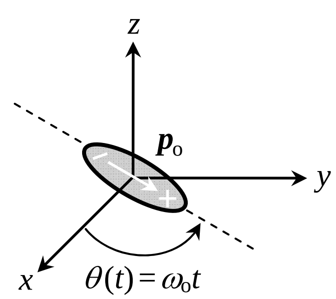
2 Pts a) Using delta-function notation, write expressions for the charge and current densities.

3 Pts b) Fourier transform the above charge and current densities, and find expressions for the scalar and vector potentials in the Fourier domain.

3 Pts c) Perform an inverse Fourier transform on the potentials in order to bring them back into the space-time domain.

2 Pts d) Find the electromagnetic fields $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$ of the moving point-charge.

Problem 3. An electric point-dipole of constant strength $\boldsymbol{p}_{\text{o}}$ is placed at the origin of a Cartesian coordinate system. With its dipole moment laying in the xy-plane, the dipole rotates around the z-axis at a constant angular velocity ω_{o}.



2 Pts a) Use delta-function notation to express the polarization density $\boldsymbol{P}(\boldsymbol{r},t)$ of the spinning dipole.

3 Pts b) Determine the bound electric charge and current densities $\rho_{\text{b}}^{(\text{e})}(\boldsymbol{r},t)$ and $\boldsymbol{J}_{\text{b}}^{(\text{e})}(\boldsymbol{r},t)$.

2 Pts c) Show that the charge and current densities obtained in part (b) satisfy the continuity equation.

3 Pts d) Find the scalar and vector potentials $\psi(\boldsymbol{r},t)$ and $\boldsymbol{A}(\boldsymbol{r},t)$ produced by the spinning dipole.

Hint: In the case of an electric point-dipole $p_{\text{o}}\cos(\omega_{\text{o}}t)\hat{\boldsymbol{z}}$ aligned with the z-axis, Problem 17 has shown that the scalar and vector potentials in a spherical coordinate system are

$$\psi(\boldsymbol{r},t) = (p_{\text{o}}\cos\theta/4\pi\varepsilon_{\text{o}}r^2)\{\cos[\omega_{\text{o}}(t-r/c)] - (\omega_{\text{o}}r/c)\sin[\omega_{\text{o}}(t-r/c)]\},$$

$$\boldsymbol{A}(\boldsymbol{r},t) = -(\mu_{\text{o}}p_{\text{o}}\omega_{\text{o}}\hat{\boldsymbol{z}}/4\pi r)\sin[\omega_{\text{o}}(t-r/c)].$$

You may use these results in answering part (d) of the problem.
