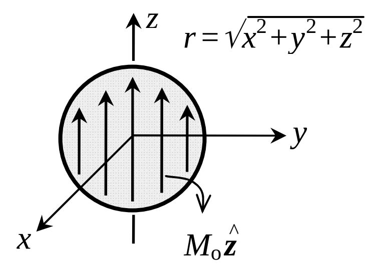
Opti 501 **Midterm Exam** (10/14/2010) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A uniformly-magnetized sphere of radius R and magnetization $M_o\hat{\boldsymbol{z}}$ is centered at the origin of the coordinate system.



4 Pts a) Find the bound magnetic charge-density $\rho_{\text{bound}}^{(m)}(\boldsymbol{r},t)$ of the magnetized sphere.

4 Pts b) Determine the bound electric current-density $\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t)$ of the magnetized sphere.

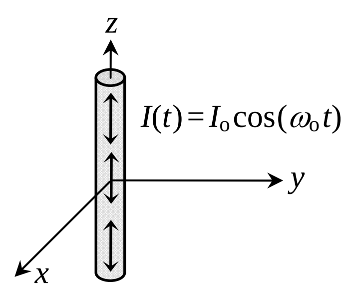
4 Pts c) What is the four-dimensional (4D) Fourier transform of $\boldsymbol{M}(\boldsymbol{r},t)$?

Hint: Define the unit-sphere function $\text{Sphere}(r)=1.0$ when $r=\sqrt{x^2+y^2+z^2}<1.0$, and 0.0 when $r>1.0$. Also, use the fact that, in a spherical coordinate system, $\hat{\boldsymbol{z}}=(\cos\theta)\hat{\boldsymbol{r}}-(\sin\theta)\hat{\boldsymbol{\theta}}$. Carry out the Divergence, Curl, and Fourier transform operations in the spherical coordinate system.

Problem 2) A cylindrical wave emanating from an infinitely long, thin wire, carrying a current $I_o\cos(\omega_o t)$ along the z-axis, has the following E- and H-fields in the cylindrical coordinate system (ρ,ϕ,z):

$$\boldsymbol{E}(\boldsymbol{r},t)=-\tfrac{1}{4}\mu_o I_o\omega_o\,[J_0(\rho\omega_o/c)\cos(\omega_o t)+Y_0(\rho\omega_o/c)\sin(\omega_o t)]\,\hat{\boldsymbol{z}}.$$

$$\boldsymbol{H}(\boldsymbol{r},t)=\frac{I_o\omega_o}{4c}\,[J_1(\rho\omega_o/c)\sin(\omega_o t)-Y_1(\rho\omega_o/c)\cos(\omega_o t)]\,\hat{\boldsymbol{\phi}}.$$



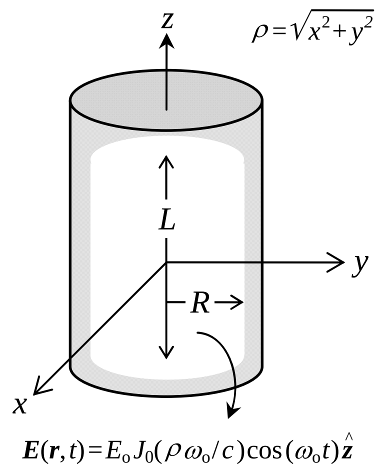
4 Pts a) Using the large-argument approximate form of the Bessel functions $J_0(\cdot)$, $J_1(\cdot)$, $Y_0(\cdot)$, and $Y_1(\cdot)$, find the electromagnetic field distribution in the far field (i.e., $\rho\gg\lambda_o$, where $\lambda_o=2\pi c/\omega_o$ is the vacuum wavelength).

4 Pts b) Determine the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ in the far field.

3 Pts c) Find the time-averaged Poynting vector, $\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle$, and show that the time-averaged energy leaving a cylinder of radius $R\gg\lambda_o$ that surrounds the radiating wire does not depend on the radius R of the cylinder.

Continued on the reverse side …

Problem 3) An electromagnetic wave is trapped within an empty, perfectly-conducting, cylindrical canister. The cavity (i.e., the hollow interior of the canister) has radius R and length L, as shown. The E-field of the trapped mode is given by $\boldsymbol{E}(\rho,\phi,z,t) = E_o J_0(\rho\omega_o/c)\cos(\omega_o t)\hat{\boldsymbol{z}}$, where $J_0(\cdot)$ is a Bessel function of the first kind, 0^{th} order, ω_o is the oscillation frequency, ρ is the radial distance from the z-axis, and c is the speed of light in vacuum.



2 Pts a) Considering that, at the walls of the cavity, the tangential component of the E-field must vanish, identify the relation between R, ω_o, and the zeros x_1, x_2, x_3, … of $J_0(x)$.

3 Pts b) Using Maxwell's third equation, $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$, determine the H-field distribution throughout the cavity. (Ignore the constant of integration.)

3 Pts c) Verify that the above E- and H-fields satisfy the remaining Maxwell's equations inside the cavity.

3 Pts d) Use Maxwell's boundary conditions to determine the distributions of surface-charge-density $\sigma_s(\boldsymbol{r},t)$ and surface-current-density $\boldsymbol{J}_s(\boldsymbol{r},t)$ on the interior walls of the cavity, i.e., the internal cylindrical surface as well as the internal flat surfaces at the top and bottom of the canister.

1 Pt e) Verify that the charge-current continuity equation, $\boldsymbol{\nabla}\cdot\boldsymbol{J}_s(\boldsymbol{r},t) + \partial\sigma_s(\boldsymbol{r},t)/\partial t = 0$, is satisfied on all the interior surfaces.
