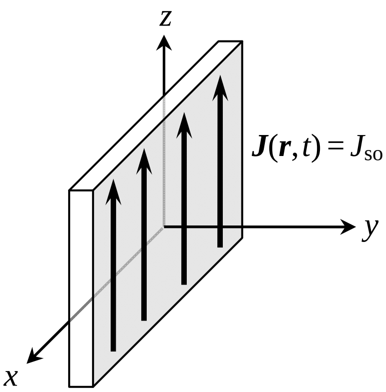
Opti 501 **2nd Midterm Exam** (10/25/2011) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) An infinitely large, thin sheet, located in the *xz*-plane, carries a constant, uniform surface-current density $J_{so}\hat{\boldsymbol{z}}$, as shown. There is no electric charge anywhere in the system, that is $\rho(\boldsymbol{r},t)=0$.



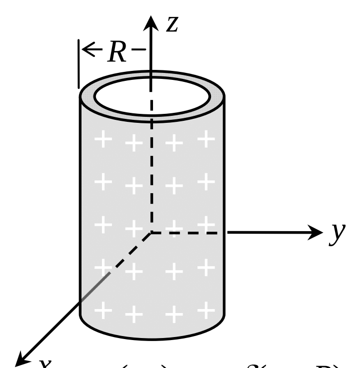
$$\boldsymbol{J}(\boldsymbol{r},t)=J_{so}\delta(y)\,\hat{\boldsymbol{z}}$$

5 Pts a) Use Ampere's law in conjunction with symmetry considerations and Maxwell's 4th equation ($\boldsymbol{\nabla}\cdot\boldsymbol{B}=0$) to determine the *H*-field in the surrounding space.

7 Pts b) Use the Fourier transform method to arrive at the same answer as found in part (a).

Hint: $\mathscr{F}\{\text{sign}(y)\}=-2\mathrm{i}/k_y$.

Problem 2) An infinitely long, thin, hollow cylinder of radius R is charged with a constant, uniform electric surface-charge density σ_{so}. There is no electric current anywhere in the system, that is $\boldsymbol{J}(\boldsymbol{r},t)=0$.



$$\rho(\boldsymbol{r},t)=\sigma_{so}\delta(\rho-R)$$

5 Pts a) Use Gauss's law in conjunction with symmetry considerations and Maxwell's 3rd equation ($\boldsymbol{\nabla}\times\boldsymbol{E}=0$) to determine the *E*-field both inside and outside the cylinder.

8 Pts b) Use the Fourier transform method to arrive at the same answer as found in part (a).

Hint: $\int_0^{2\pi}\exp(\pm\mathrm{i}\beta\cos\phi)\,\mathrm{d}\phi=2\pi J_0(\beta);$ G & R **3.915**-2

$$\int_0^{2\pi}\cos\phi\exp(\pm\mathrm{i}\beta\cos\phi)\,\mathrm{d}\phi=\pm\mathrm{i}2\pi J_1(\beta);$$ G & R **3.915**-2

$$\int_0^{\infty}J_0(ax)\,J_1(bx)\,\mathrm{d}x=\begin{cases}1/b; & a<b,\\ 1/(2b); & a=b,\\ 0; & a>b.\end{cases}$$ G & R **6.512**-3

Do not confuse the symbol ρ used for charge-density with the same symbol used to represent radial distance in the cylindrical coordinate system.
