Opti 501 **Midterm Exam** (11/12/2009) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1. A monochromatic, *homogeneous* plane-wave propagates in free space along the direction $\hat{\boldsymbol{k}}$ of its real-valued k-vector. The electric and magnetic fields are given by

$$\boldsymbol{E}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{E}_\mathrm{o}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\},$$

$$\boldsymbol{H}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{H}_\mathrm{o}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}.$$

In general, $\boldsymbol{E}_\mathrm{o}$ and $\boldsymbol{H}_\mathrm{o}$ are complex-valued.

1 pt a) When is the plane-wave linearly-polarized?

1 pt b) When is the plane-wave circularly-polarized?

1 pt c) What is the relation between k and ω?

1 pt d) What is the relation between $\boldsymbol{k}$ and the real and imaginary components $\boldsymbol{E}_\mathrm{o}'$ and $\boldsymbol{E}_\mathrm{o}''$ of $\boldsymbol{E}_\mathrm{o}$?

1 pt e) What is the relation between $\boldsymbol{k}$ and the real and imaginary components $\boldsymbol{H}_\mathrm{o}'$ and $\boldsymbol{H}_\mathrm{o}''$ of $\boldsymbol{H}_\mathrm{o}$?

1 pt f) How is $\boldsymbol{H}_\mathrm{o}$ related to $\boldsymbol{k}$ and $\boldsymbol{E}_\mathrm{o}$? How are the magnitudes of $\boldsymbol{H}_\mathrm{o}', \boldsymbol{H}_\mathrm{o}''$ related to those of $\boldsymbol{E}_\mathrm{o}', \boldsymbol{E}_\mathrm{o}''$?

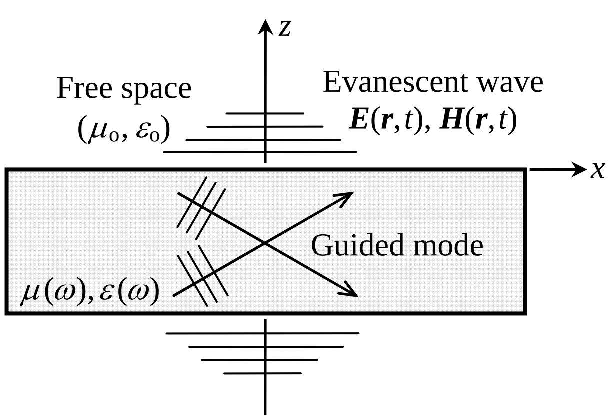
2 pts g) Write an expression for the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ of the plane-wave in terms of $\hat{\boldsymbol{k}}$, $\boldsymbol{E}_\mathrm{o}', \boldsymbol{E}_\mathrm{o}''$, ω, $c=1/\sqrt{\mu_\mathrm{o}\varepsilon_\mathrm{o}}$ and $Z_\mathrm{o}=\sqrt{\mu_\mathrm{o}/\varepsilon_\mathrm{o}}$. In what direction does the energy propagate in 3D space?

Problem 2. An *evanescent*, monochromatic, plane-wave resides in the region above (and also below) a single-mode dielectric slab waveguide, as shown. The electromagnetic fields of the evanescent wave in the half-space $z>0$ are given by

$$\boldsymbol{E}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{E}_\mathrm{o}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\},$$

$$\boldsymbol{H}(\boldsymbol{r},t)=\mathrm{Re}\{\boldsymbol{H}_\mathrm{o}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}.$$

In this problem $\boldsymbol{k}=k_x\hat{\boldsymbol{x}}+\mathrm{i}k_z\hat{\boldsymbol{z}}$, $\boldsymbol{E}_\mathrm{o}=E_{x\mathrm{o}}\hat{\boldsymbol{x}}+\mathrm{i}E_{z\mathrm{o}}\hat{\boldsymbol{z}}$, and $\boldsymbol{H}_\mathrm{o}=\mathrm{i}H_{y\mathrm{o}}\hat{\boldsymbol{y}}$, with k_x, k_z, $E_{x\mathrm{o}}$, $E_{z\mathrm{o}}$, and $H_{y\mathrm{o}}$ all real-valued.



1 pt a) What is the relation among k_x, k_z, and ω/c?

1 pt b) How are k_x, k_z, $E_{x\mathrm{o}}$ and $E_{z\mathrm{o}}$ related?

2 pts c) How is $H_{y\mathrm{o}}$ related to $k_x, \omega, c, Z_\mathrm{o}$ and $E_{x\mathrm{o}}$?

2 pts d) Write an expression for the time-averaged Poynting vector $\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle$ in terms of k_x, ω, c, Z_o and $E_{x\mathrm{o}}$. In what direction does the evanescent-field's energy flow?

Problem 3. A monochromatic, *homogeneous*, plane-wave propagates along the direction of its real-valued k-vector within an isotropic, homogeneous, linear medium specified by its dielectric permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$. Both $\varepsilon(\omega)$ and $\mu(\omega)$ are real and positive at the frequency of interest, ω. The electric and magnetic fields are $\boldsymbol{E}(\boldsymbol{r},t)=\text{Re}\{\boldsymbol{E}_o\exp[\text{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}$ and $\boldsymbol{H}(\boldsymbol{r},t)=\text{Re}\{\boldsymbol{H}_o\exp[\text{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}$, where, in general, $\boldsymbol{E}_o$ and $\boldsymbol{H}_o$ are complex-valued.

1 pt a) What is the relation between k, ω, c, and the material parameters?

1 pt b) What is the relation between $\boldsymbol{k}$ and the real and imaginary components $\boldsymbol{E}_o'$ and $\boldsymbol{E}_o''$ of $\boldsymbol{E}_o$?

1 pt c) What is the relation between $\boldsymbol{k}$ and the real and imaginary components $\boldsymbol{H}_o'$ and $\boldsymbol{H}_o''$ of $\boldsymbol{H}_o$?

1 pt d) How is $\boldsymbol{H}_o$ related to $\boldsymbol{k}$, $\boldsymbol{E}_o$, and the material parameters? How are the magnitudes of $\boldsymbol{H}_o', \boldsymbol{H}_o''$ related to those of $\boldsymbol{E}_o', \boldsymbol{E}_o''$, and the material parameters?

2 pts e) Write an expression for the Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ of the plane-wave in terms of $\hat{\boldsymbol{k}}$, $\boldsymbol{E}_o', \boldsymbol{E}_o'', \omega$, $\varepsilon(\omega)$, $\mu(\omega)$, $c=1/\sqrt{\mu_o\varepsilon_o}$, and $Z_o=\sqrt{\mu_o/\varepsilon_o}$. Specify the direction of $\boldsymbol{S}(\boldsymbol{r},t)$ in 3D space.

Problem 4. The scalar and vector potentials of a plane-wave residing in an isotropic, homogeneous, linear medium devoid of free charges and free currents [i.e., $\rho_{\text{free}}(\boldsymbol{r},t)=0$ and $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)=0$] are given by

$$\psi(\boldsymbol{r},t)=\text{Re}\{\psi_o\exp[\text{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\},$$

$$\boldsymbol{A}(\boldsymbol{r},t)=\text{Re}\{\boldsymbol{A}_o\exp[\text{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}.$$

The medium is specified by its dielectric permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$, both of which are real-valued and positive at the frequency of interest, ω.

1 pt a) Find the E-field of the plane-wave in terms of ψ_o, $\boldsymbol{A}_o$, $\boldsymbol{k}$, and ω.

1 pt b) Find the H-field of the plane-wave in terms of $\boldsymbol{A}_o$, $\boldsymbol{k}$, ω, and the material parameters.

2 pts c) Under what circumstances will the fields obtained in (a) and (b) above satisfy all four equations of Maxwell?

1 pt d) Do the scalar and vector potentials specified in this problem satisfy the Lorenz gauge?
