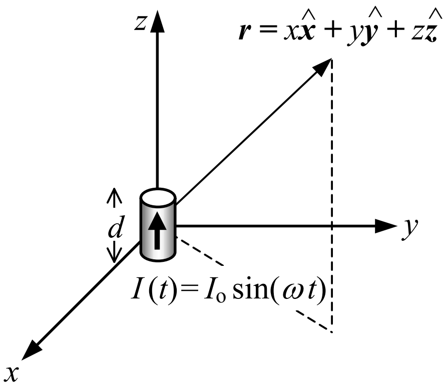
Opti 501 **Midterm Exam** (11/15/2005) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple the pages together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

1) For the short dipole oscillator shown in the figure:



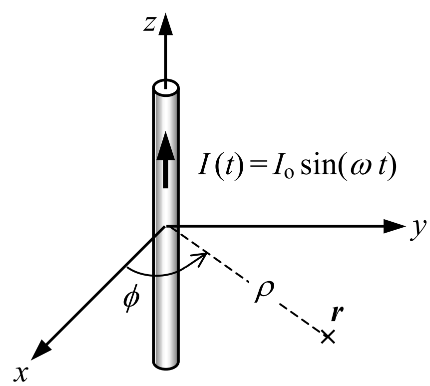
(2 pts) a) Write down the expression for the vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$.

(3 pts) b) Derive the scalar potential $\psi(\boldsymbol{r}, t)$ using the relationship $\nabla \cdot \boldsymbol{A}(\boldsymbol{r}, t) + (1/c^2)\, \partial\psi(\boldsymbol{r}, t)/\partial t = 0$.

(In the class, $\psi(\boldsymbol{r}, t)$ was derived by considering the charges at the top and bottom of the oscillator.)

2) The thin, infinitely long wire shown in the figure carries the current $I(t) = I_o \sin(\omega t)$ along the z-axis. The vector potential $\boldsymbol{A}$ in the space surrounding the wire is given by:

$$\boldsymbol{A}(\boldsymbol{r}, t) = -\tfrac{1}{4}\,\mu_o I_o \left[Y_o(k_o\rho)\sin(\omega t) + J_o(k_o\rho)\cos(\omega t)\right]\hat{\boldsymbol{z}}.$$

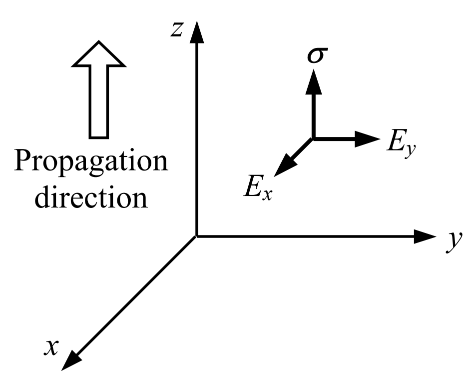


(2 pts) a) Use the limiting form of the Bessel functions $Y_o(x)$, $J_o(x)$ for small x to write an expression for $\boldsymbol{A}(\boldsymbol{r}, t)$ in the limit when the radial distance ρ is small (i.e., in the near-field region of the wire).

(3 pts) b) Using the approximate expression for $\boldsymbol{A}(\boldsymbol{r}, t)$ found in (a), and the relation $\boldsymbol{B}(\boldsymbol{r}, t) = \nabla\times\boldsymbol{A}(\boldsymbol{r}, t)$, find the magnetic-field distribution $\boldsymbol{H}(\boldsymbol{r}, t)$ in the near-field region. Show that your result is consistent with Ampere's law, $\nabla\times\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{J}(\boldsymbol{r}, t)$.

(2 pts) c) Use the relation $\boldsymbol{E}(\boldsymbol{r}, t) = -\nabla\psi(\boldsymbol{r}, t) - \partial\boldsymbol{A}(\boldsymbol{r}, t)/\partial t$ to find the electric-field distribution $\boldsymbol{E}(\boldsymbol{r}, t)$ in the near-field region.

3) A homogeneous plane wave propagates in free space along the z-axis. The oscillation frequency is $\omega = 2\pi f$, the wavelength is $\lambda_o = c/f$, the propagation constant is $k_o = 2\pi/\lambda_o$, the speed of light is c, and the impedance of the free space is Z_o. The only restrictions on the fields are those imposed by Maxwell's equations.



(2 pts) a) Write expressions for the propagation vector $\boldsymbol{\sigma}$, the E-field amplitude $\boldsymbol{E}_o = E_{xo}\hat{\boldsymbol{x}} + E_{yo}\hat{\boldsymbol{y}} + E_{zo}\hat{\boldsymbol{z}}$, and the H-field amplitude $\boldsymbol{H}_o = H_{xo}\hat{\boldsymbol{x}} + H_{yo}\hat{\boldsymbol{y}} + H_{zo}\hat{\boldsymbol{z}}$, consistent with Maxwell's equations.

(1 pt) b) What conditions should E_{xo} and E_{yo} satisfy for the plane-wave to be linearly polarized?

(1 pt) c) What conditions should E_{xo} and E_{yo} satisfy for the plane-wave to be circularly polarized?

(2 pts) d) Let $E_{xo} = |E_{xo}|\exp(i\phi_{xo})$ and $E_{yo} = |E_{yo}|\exp(i\phi_{yo})$. Assuming $|E_{xo}| > |E_{yo}|$ and $\phi_{xo} - \phi_{yo} = 90°$, what is the polarization ellipticity η of the plane-wave?

(2 pts) e) Starting from the formula $<\boldsymbol{S}(\boldsymbol{r}, t)> = \frac{1}{2} Re(\boldsymbol{E} \times \boldsymbol{H}^*)$ and showing every step of the calculation, derive an expression for the time-averaged Poynting vector $<\boldsymbol{S}(\boldsymbol{r}, t)>$ in terms of E_{xo} and E_{yo}.

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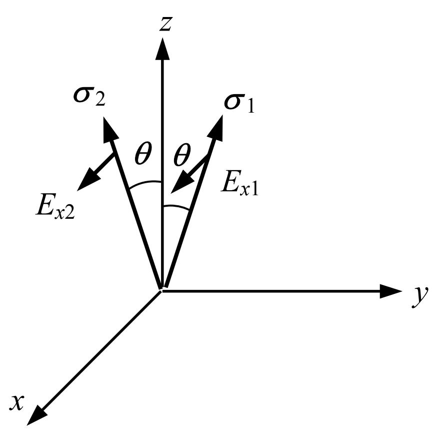
4) Two homogeneous plane-waves propagate in free space along the directions $\boldsymbol{\sigma}_1 = \sin\theta\,\hat{\boldsymbol{y}} + \cos\theta\,\hat{\boldsymbol{z}}$ and $\boldsymbol{\sigma}_2 = -\sin\theta\,\hat{\boldsymbol{y}} + \cos\theta\,\hat{\boldsymbol{z}}$, as shown. Both plane-waves are linearly polarized along the x-axis with amplitudes $E_{x1} = |E_{x1}|\exp(i\phi_{x1})$ and $E_{x2} = |E_{x2}|\exp(i\phi_{x2})$.



(2 pts) a) For each plane-wave write expressions for the electric and magnetic fields $\boldsymbol{E}(\boldsymbol{r}, t)$ and $\boldsymbol{H}(\boldsymbol{r}, t)$ in terms of the corresponding E-field amplitudes, the orientation angle θ, the propagation constant k_o, the speed of light c, and the impedance Z_o of the free space.

(3 pts) b) Compute the time-averaged Poynting vector $<\boldsymbol{S}(\boldsymbol{r}, t)>$ for the superposition of the two plane-waves. Explain how interference between the two plane-waves affects the behavior of $<\boldsymbol{S}(\boldsymbol{r}, t)>$.
