Opti 501 **1^{st} Midterm Exam** (9/27/2011) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

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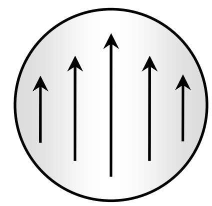
8 Pts **Problem 1)** A uniformly-magnetized sphere of radius R and magnetization $M_o\hat{\boldsymbol{z}}$ is shown in the figure. Using the *conceptual* definitions of the divergence and curl operators, find the density of bound magnetic charge $\rho^{(m)}_{\text{bound}}$ and also the density of bound electric current $\boldsymbol{J}^{(e)}_{\text{bound}}$ on the surface of the sphere. (Recall that the proper symbols for surface charge and current densities are σ_s and $\boldsymbol{J}_s$.)

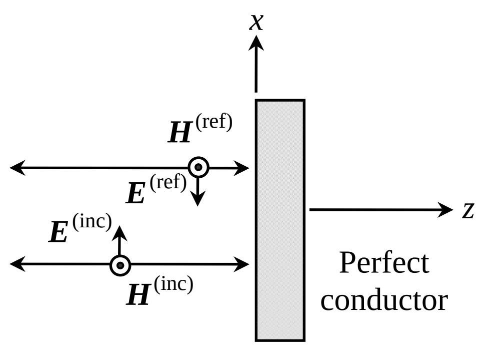


$\boldsymbol{M}(\boldsymbol{r}) = M_o\hat{\boldsymbol{z}}$; $r \le R$

Problem 2) A monochromatic plane-wave of frequency ω traveling in free space is reflected at normal incidence from the flat surface of a perfect conductor. Denoting the speed of light in vacuum by $c = 1/\sqrt{\mu_o\varepsilon_o}$ and the impedance of free space by $Z_o = \sqrt{\mu_o/\varepsilon_o}$, the incident and reflected E- and H-fields are given by

$$\begin{cases} \boldsymbol{E}^{(\text{inc})}(\boldsymbol{r},t) = E_o\hat{\boldsymbol{x}}\cos[(\omega/c)z - \omega t], \\ \boldsymbol{H}^{(\text{inc})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\cos[(\omega/c)z - \omega t]. \end{cases}$$

$$\begin{cases} \boldsymbol{E}^{(\text{ref})}(\boldsymbol{r},t) = -E_o\hat{\boldsymbol{x}}\cos[(\omega/c)z + \omega t], \\ \boldsymbol{H}^{(\text{ref})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\cos[(\omega/c)z + \omega t]. \end{cases}$$



2 Pts a) Write expressions for the total E-field and total H-field amplitudes in the half-space $z \le 0$.

Hint: $\cos a + \cos b = 2\cos[(a+b)/2]\cos[(a-b)/2]$; $\cos a - \cos b = -2\sin[(a+b)/2]\sin[(a-b)/2]$.

2 Pts b) Identify locations along the z-axis where the E-field is exactly equal to zero, and also locations where the H-field is exactly zero.

2 Pts c) Determine the local energy densities of the E- and H-fields in the half-space $z \le 0$.

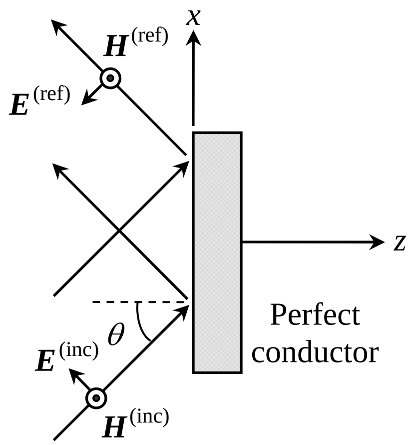
3 Pts d) Find the total Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ in the half-space $z \le 0$, and explain the behavior of the electromagnetic energy as a function of time by analyzing the time-dependence of the Poynting vector in relation to the local energy densities of the E- and H-fields.

Hint: $2\sin(a)\cos(a) = \sin(2a)$.

Problem 3) A monochromatic plane-wave of frequency ω traveling in free space is reflected from a perfectly-conducting, flat mirror surface at an oblique incidence angle θ. In the half-space $z \le 0$, the electric and magnetic fields of the incident and reflected waves are given by

$$\begin{cases} \boldsymbol{E}^{(\text{inc})}(\boldsymbol{r},t) = E_o(\cos\theta\hat{\boldsymbol{x}} - \sin\theta\hat{\boldsymbol{z}})\exp\{\mathrm{i}(\omega/c)[(\sin\theta)x + (\cos\theta)z - ct]\}, \\ \boldsymbol{H}^{(\text{inc})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\exp\{\mathrm{i}(\omega/c)[(\sin\theta)x + (\cos\theta)z - ct]\}. \end{cases}$$

$$\begin{cases} \boldsymbol{E}^{(\text{ref})}(\boldsymbol{r},t) = -E_o(\cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{z}})\exp\{\mathrm{i}(\omega/c)[(\sin\theta)x - (\cos\theta)z - ct]\}, \\ \boldsymbol{H}^{(\text{ref})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\exp\{\mathrm{i}(\omega/c)[(\sin\theta)x - (\cos\theta)z - ct]\}. \end{cases}$$



2 Pts a) Find the tangential component of the E-field at the mirror surface, and verify that it satisfies the relevant boundary condition.

2 Pts b) Find the tangential component of the H-field at the mirror surface, and determine the current density $\boldsymbol{J}_s(x,y,z=0,t)$ that must exist on the surface in order to satisfy the relevant boundary condition.

2 Pts c) Find the perpendicular component of the E-field at the mirror surface, and determine the charge density $\sigma_s(x,y,z=0,t)$ that must exist on the surface in order to satisfy the relevant boundary condition.

2 Pts d) Show that the surface charge and current densities obtained in parts (b) and (c) satisfy the continuity equation $\boldsymbol{\nabla}\cdot\boldsymbol{J}_s + \partial\sigma_s/\partial t = 0$.
