Opti 501 **Midterm Exam** (10/7/2008) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together. Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

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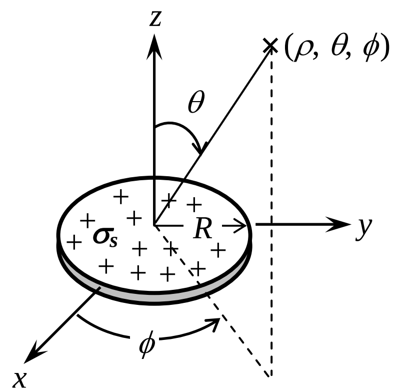
1. A non-conducting (e.g., plastic) disk of radius R and negligible thickness is uniformly charged with a constant surface charge density σ_s. In the spherical coordinate system shown in the figure, the scalar potential is $\psi(\rho, \theta, \phi)$.



2 pts a) Given the symmetry of the system, without any calculations, what can one say about $\psi(\rho, \theta, \phi)$?

2 pts b) Considering the form of the scalar potential determined in part (a) above, what can one say about the various E-field components, $E_\rho(\rho, \theta, \phi)\hat{\boldsymbol{\rho}} + E_\theta(\rho, \theta, \phi)\hat{\boldsymbol{\theta}} + E_\phi(\rho, \theta, \phi)\hat{\boldsymbol{\phi}}$?

1 pt c) At an arbitrary point located within the xy-plane but outside the disk, i.e., $(\rho > R, \theta = \frac{1}{2}\pi, \phi)$, which components of the E-field, if any, are equal to zero?

2 pts d) What is the magnitude of E_θ at a point A slightly above the disk, i.e., $(\rho < R, \theta = \frac{1}{2}\pi^-, \phi)$, or at a point B slightly below the disk, i.e., $(\rho < R, \theta = \frac{1}{2}\pi^+, \phi)$?

1 pt e) Without any calculations, what can one say about E_ρ at the points A and B mentioned in part (d) above?

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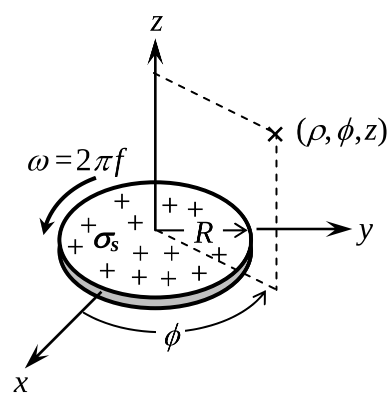
2. The disk described in Problem 1 now rotates around the z-axis at a constant angular velocity $\omega = 2\pi f$. As the disk is non-conducting, its charges are immobile and, therefore, its surface charge density σ_s remains the same as that of the stationary disk.



1 pt a) Is this an electro-static problem, a magneto-static problem, or both? Are the scalar potential ψ and the E-field distribution any different than those determined in Problem 1?

1 pt b) What is the surface current density $\boldsymbol{J}_s(\rho)$ of the disk?

1 pt c) Is the continuity equation $\nabla \cdot \boldsymbol{J}_s + \partial\sigma_s/\partial t = 0$ satisfied?

1 pt d) Expressing the vector potential in cylindrical coordinates as $\boldsymbol{A}(\rho, \phi, z)$, given the symmetry of the problem, what can one say about $\boldsymbol{A}(\rho, \phi, z)$ without any calculations?

1 pt e) Considering the form of the vector potential determined in part (d) above, what can one say about the various H-field components, $H_\rho(\rho, \phi, z)\hat{\boldsymbol{\rho}} + H_\phi(\rho, \phi, z)\hat{\boldsymbol{\phi}} + H_z(\rho, \phi, z)\hat{\boldsymbol{z}}$?

1 pt f) At an arbitrary point located within the xy-plane but outside the disk, i.e., $(\rho > R, \phi, z = 0)$, which components of the H-field, if any, are equal to zero?

1 pt g) What is the magnitude of H_ρ at a point A slightly above the disk, i.e., $(\rho < R, \phi, z = 0^+)$, or at a point B slightly below the disk, i.e., $(\rho < R, \phi, z = 0^-)$?

1 pt h) Without any calculations, what can one say about H_z at the points A and B mentioned in part (g) above?

1 pt i) What is the magnetic dipole moment $\boldsymbol{m}$ of the rotating disk?

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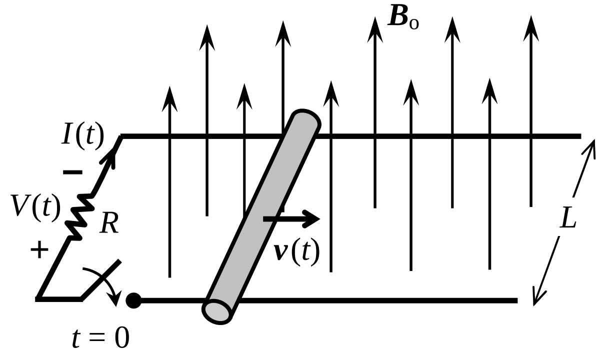
3. A high-conductivity metallic rod of length L and mass M moves with constant velocity $\boldsymbol{v}_o$ over an open circuit, as shown. In addition to a light bulb (resistance = R), the circuit contains a switch, which closes at $t = 0$. Crossing the circuit is a uniform, time-independent magnetic field $\boldsymbol{B}_o$, perpendicular to the plane of the circuit at each and every point.



2 pts a) Describe (in words) what happens to the light bulb and the rod after the switch is closed, i.e., at $t = 0$ and beyond.

2 pts b) The rod slows down after the switch is closed. Denoting its velocity by $\boldsymbol{v}(t)$ for $t \geq 0$, write expressions for the voltage $V(t)$ and current $I(t)$ of the light bulb in terms of the length L of the rod, its velocity $\boldsymbol{v}(t)$, the magnetic field strength B_o, and the resistance R.

2 pts c) Using the Lorentz law of force, $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$, express the braking force on the rod in terms of B_o, L, and the current $I(t)$ flowing in the rod. Using Newton's law of motion, $\boldsymbol{F}(t) = M\,\mathrm{d}\boldsymbol{v}(t)/\mathrm{d}t$, and the relation between $\boldsymbol{v}(t)$ and $I(t)$ found in part (b), determine the rod's velocity $\boldsymbol{v}(t)$ for $t \geq 0$. [**Note**: the velocity of the rod immediately after the closing of the switch is $\boldsymbol{v}(t = 0^+) = \boldsymbol{v}_o$].

2 pts d) Considering that the instantaneous power delivered to the light bulb is $P(t) = V(t)I(t)$, show that, between $t = 0$ and $t = \infty$ (when the rod comes to a halt), the total energy consumed by the light bulb is equal to the rod's initial kinetic energy $\frac{1}{2}Mv_o^2$.
