Opti 501 **Midterm Exam** (10/4/2007) **Time: 75 minutes**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

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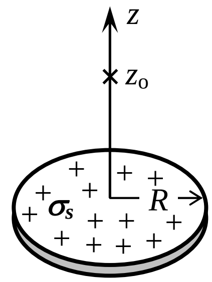
1. A disk of radius R, uniform surface charge density σ_s, and negligible thickness, is centered on the z-axis, as shown.



1 pt a) Use Coulomb's law to determine the electric field $\boldsymbol{E}$ on the z-axis as a function of the distance z_o and the radius R.

1 pt b) Find the scalar potential ψ along the z-axis as a function of z_o and R.

2 pts c) Using the scalar potential found in (b), determine the E-field along the z-axis. Verify that your result agrees with the one obtained in part (a).

1 pt d) What is the strength of the E-field at $z = z_o$ when $R \to \infty$?

1 pt e) For a given z_o, what is the minimum value of R that brings the strength of the E-field at $z = z_o$ to within 1% of the limiting value found in (d)?

1 pt f) In terms of the total charge Q of the disk, find the E-field at $z = z_o$ when $R \to 0$.

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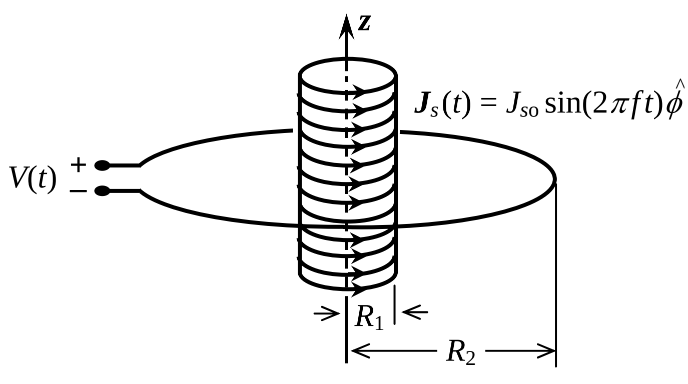
2. An infinitely long solenoid of radius R_1 carries a surface current density $\boldsymbol{J}_s(t) = J_{so}\sin(2\pi f t)\hat{\phi}$, as shown in the figure. The oscillation frequency f is fairly small (i.e., fields vary slowly with time). A circular loop of conducting wire (radius $= R_2 > R_1$) is placed around the solenoid. The loop and the cylinder are both centered on the z-axis.

2 pts a) Determine the magnetic fields $\boldsymbol{H}(t)$ and $\boldsymbol{B}(t)$ inside the solenoid.

2 pts b) Using Faraday's law, determine the induced voltage $V(t)$ in the external loop.

2 pts c) In one of the homework assignments, you calculated the vector potential $\boldsymbol{A}(\boldsymbol{r})$ both inside and outside a solenoid. Can you explain the existence of the voltage $V(t)$ on the basis of the vector potential acting directly on the conducting loop?



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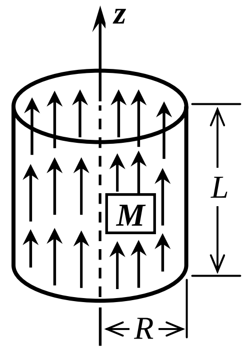
3. A cylindrical permanent magnet is uniformly magnetized along the z-axis, as shown. The magnetization of the material (i.e., strength of atomic dipole moments per unit volume) is $\boldsymbol{M}(\boldsymbol{r}) = M_o\hat{\boldsymbol{z}}$. In general, the *magnetic charge density* is defined as $\rho_m = -\nabla\cdot\boldsymbol{M}$, while the *magnetic current density* is $\boldsymbol{J}_m = \nabla\times\boldsymbol{M}$. Considering that $\boldsymbol{M}$ is uniform within the cylinder, the only place where these magnetic charges and currents can exist is at the cylinder's various surfaces. Therefore, this permanent magnet can only have surface magnetic charge and current densities, σ_{ms} and $\boldsymbol{J}_{ms}$.



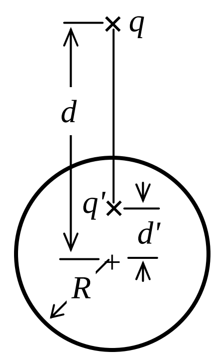
2 pts a) Using the definitions of divergence and curl, determine σ_{ms} and $\boldsymbol{J}_{ms}$.

2 pts b) Considering that $\boldsymbol{B} = \mu_o(\boldsymbol{H} + \boldsymbol{M})$ and that, according to Maxwell's 4th equation $\nabla\cdot\boldsymbol{B} = 0$, show that, in magnetostatics in the absence of free currents $\boldsymbol{J}$, the H-field can be determined from ρ_m (or σ_{ms}) in the same way that the displacement field $\boldsymbol{D}$ of electrostatics in vacuum is determined from the electric charge distribution ρ_e (or σ_{es}). For our cylindrical permanent magnet, draw the lines of the H-field both inside and outside the cylinder.

2 pts c) Using the *Biot-Savart* law, write a general expression relating the B-field produced by $\boldsymbol{M}(\boldsymbol{r})$ to the magnetic current density $\boldsymbol{J}_m$. For our cylindrical permanent magnet, draw the B-field lines both inside and outside the cylinder.

In the following problem you are asked to use the *method of images* to determine the behavior of a perfect conductor in the presence of an external charge.

4. A perfectly conducting sphere of radius R is centered at a distance d from a point charge q, as shown.



2 pts a) The sphere is imagined to have been replaced with a point charge q' located a distance d' from the center of the sphere along the straight line that connects q to the sphere's center. Find q' and d' such that the scalar potential ψ at the sphere's surface is zero.

1 pt b) What is the total charge collected on the surface of the conducting sphere, if it is held at zero potential?

1 pt c) What other (fictitious) charge, q'', needs to be placed inside the sphere (and at what location) if the sphere was initially held at a non-zero potential, say, ψ_o.

1 pt d) What would be the total charge collected on the sphere's surface, if it were held at the constant potential ψ_o in the presence of an external charge q at distance d from the center?

1 pt e) Describe (in words) how one should go about calculating the surface charge density at each point on the surface of the conductor.

Hint: In the triangle shown, $r = \sqrt{R^2 + d^2 - 2Rd\cos\theta}$.