Opti 501 **Final Exam** (6/22/2010) **Time: 2hours**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

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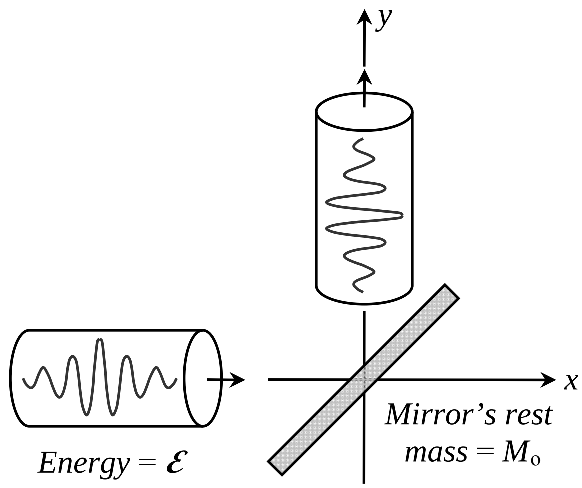
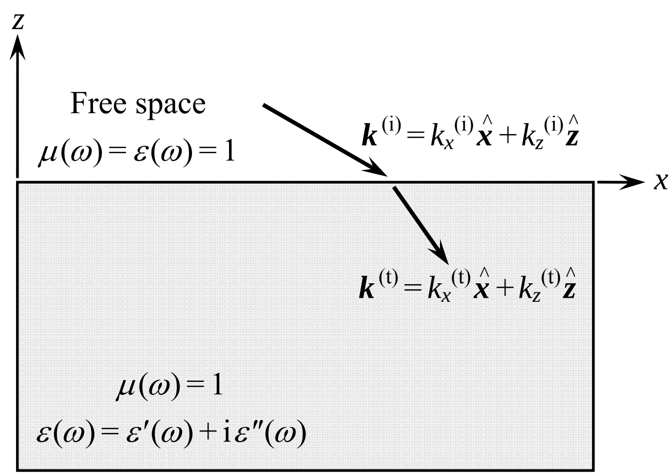
10 Pts **Problem 1**) The figure shows a light pulse of energy $\mathcal{E}$, originally propagating (in free space) along the x-axis, re-directed to propagate along the y-axis, upon reflection from a perfectly-reflecting flat mirror. The reflection should be treated as an elastic collision, in the sense that none of the light pulse's energy is absorbed by the mirror. (In other words, one may say in relativistic language that the rest mass M_o of the mirror is the same before and after collision.) Assuming the mirror has been stationary before collision, determine its mechanical momentum and energy after the light pulse has been fully reflected.



Hint: The equations may be set up and solved relativistically or non-relativistically. You do *not* need to solve the problem both ways; either solution will be acceptable.

Problem 2) Consider the flat interface between free space and a homogeneous, linear, isotropic medium specified by its relative permeability, $\mu(\omega)=1$, and permittivity, $\varepsilon(\omega)=\varepsilon'(\omega)+\mathrm{i}\varepsilon''(\omega)$. The incident and transmitted plane-waves are specified by their k-vectors, which are assumed to be confined to the xz-plane, that is, $k_y^{(\mathrm{i})}=k_y^{(\mathrm{t})}=0$. In general, both plane-waves are inhomogeneous, and we shall assume that $k_x^{(\mathrm{i})}, k_z^{(\mathrm{i})}$, $k_x^{(\mathrm{t})}$, and $k_z^{(\mathrm{t})}$ are complex-valued. The goal of this problem is to explore conditions under which a reflected beam does *not* exist.



2 Pts a) Using Snell's law and the dispersion relation for each medium, write down all the relations that exist among $k_x^{(\mathrm{i})}, k_z^{(\mathrm{i})}$, $k_x^{(\mathrm{t})}$, and $k_z^{(\mathrm{t})}$.

5 Pts b) For *each* plane-wave, assuming the E- and H-field amplitudes are $\boldsymbol{E}_o=(E_{xo}, E_{yo}, E_{zo})$ and $\boldsymbol{H}_o=(H_{xo}, H_{yo}, H_{zo})$, use Maxwell's equations to relate the various field components to each other.

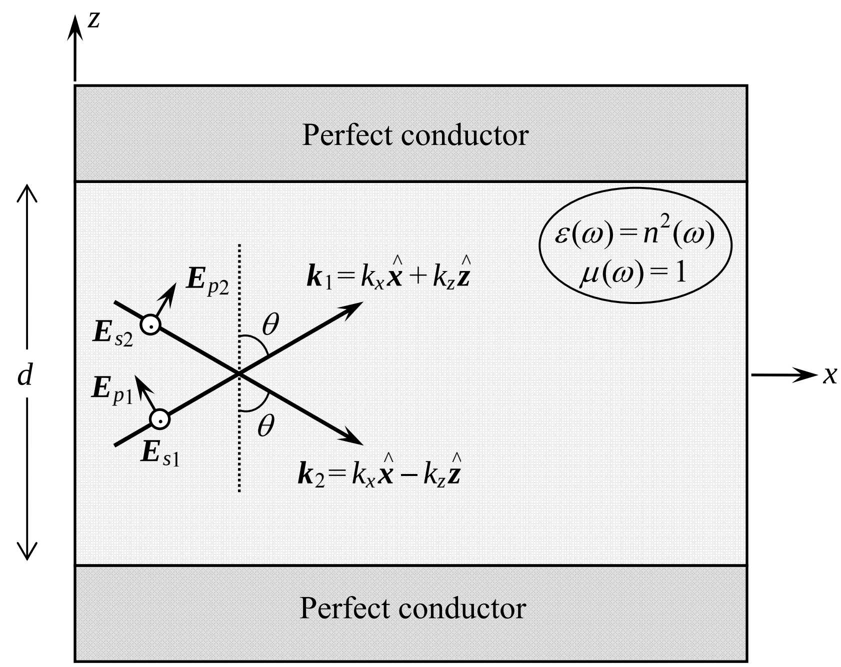
3 Pts c) At the $z=0$ interface, write the relevant boundary conditions, then relate the field components of the transmitted plane-wave to those of the incident plane-wave.

5 Pts d) Under what conditions are the boundary conditions precisely satisfied *without* the need to introduce a reflected plane-wave? For which state of polarization (p or s) is it *impossible* to find circumstances under which the Fresnel reflection coefficient would be precisely zero?

5 Pts e) Explore the conditions under which the Fresnel reflection coefficient vanishes (i.e., boundary conditions are matched without the need for a reflected plane-wave) in four special cases:

(i) $\varepsilon' > 0$, $\varepsilon'' = 0$; (ii) $\varepsilon' < -1$, $\varepsilon'' = 0$; (iii) $\varepsilon' < 0$, $\varepsilon'' > 0$; (iv) $\varepsilon' > 0$, $\varepsilon'' > 0$.

Problem 3) A slab waveguide consists of a homogeneous, isotropic, linear, transparent, non-magnetic dielectric of thickness d and (real-valued) refractive index $n(\omega)$, sandwiched between two perfect conductors, as shown in the figure. A monochromatic guided mode propagating along the x-axis may be expressed as the superposition of two homogeneous plane-waves whose k-vectors are $\boldsymbol{k}_1 = k_x\hat{\boldsymbol{x}} + k_z\hat{\boldsymbol{z}}$ and $\boldsymbol{k}_2 = k_x\hat{\boldsymbol{x}} - k_z\hat{\boldsymbol{z}}$. With reference to the figure, $k_x = (\omega/c)n(\omega)\sin\theta$.



3 Pts a) Write expressions for k_z, E_{z1}, H_{x1}, H_{y1}, H_{z1}, E_{z2}, H_{x2}, H_{y2} and H_{z2} in terms of the remaining parameters, namely, k_x, ω, θ, $n(\omega)$, E_{x1}, E_{y1}, E_{x2}, and E_{y2}.

5 Pts b) Setting $E_{x2} = E_{x1}$ and $E_{y2} = E_{y1}$, write the complete expressions for total fields, $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$ in terms of E_{x1}, E_{y1}, and the remaining parameters of the waveguide. (The resulting mode is called an *even* mode, because E_x, E_y, and H_z are symmetric with respect to the x-axis.)

5 Pts c) What restrictions do the boundary conditions at $z = \pm d/2$ impose on the mode parameters? Is there a thickness d below which one or both polarization states (p and/or s) will fail to propagate within the waveguide? If so, what is the critical thickness d below which guiding is "cut-off"? When does the waveguide become single-mode for p-polarized light? When does it become single-mode for s-polarized light?

3 Pts d) Treating p- and s-polarized modes separately, find the surface charge and current densities, $\sigma_s(x, z = d/2, t)$ and $\boldsymbol{J}_s(x, z = d/2, t)$, on the surface of the upper conductor that is in contact with the top facet of the dielectric slab.

4 Pts e) Repeat the above parts (b), (c), and (d) for *odd* modes of the wave-guide, i.e., modes for which $E_{x2} = -E_{x1}$ and $E_{y2} = -E_{y1}$. Show that there is no "cut-off" for the p-polarized odd mode.
