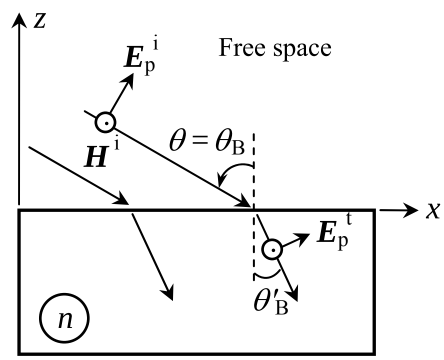
Opti 501 **Final Exam** (12/14/2010) **Time: 2 Hours**

Please write your name and ID number on all the pages, then staple them together.
Answer all the questions.

Note: Bold symbols represent vectors and vector fields.

Problem 1) A p-polarized monochromatic plane-wave is incident from free-space at the surface of a transparent, semi-infinite, dielectric medium of refractive index n. (As usual, we are assuming $\mu = 1.0$ at optical frequencies.) At the Brewster incidence angle, where $\theta = \theta_B$, the reflected beam disappears.



4 Pts a) Write expressions for the E- and H-field distributions for both the incident and transmitted beams at $\theta = \theta_B$.

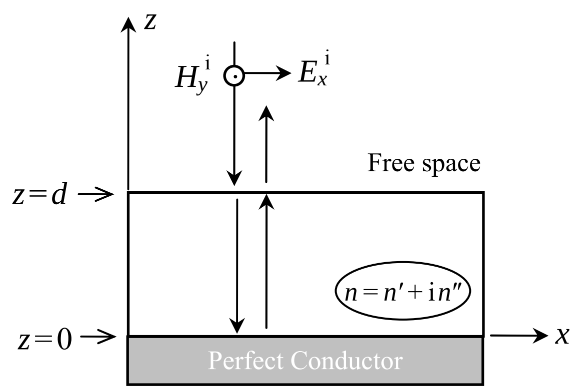
4 Pts b) Match the boundary conditions to determine the transmitted E- and H-field amplitudes in terms of the incident E-field amplitude.

3 Pts c) Verify the continuity of the perpendicular D-field at the vacuum-dielectric interface.

4 Pts d) Using the time-averaged Poynting vector $\langle \boldsymbol{S}(\boldsymbol{r}, t) \rangle$, show that the rate-of-flow of energy in the incident beam is precisely equal to that in the transmitted beam.

3 Pts e) Find the distribution of the *bound* surface-charge-density $\sigma_s^{(\text{bound})}(x, y, z = 0, t)$ at the interface between free-space and the transparent medium.

Problem 2) A linearly-polarized monochromatic plane-wave arrives from free-space at normal incidence at the surface of an absorptive dielectric. The dielectric has thickness d and refractive index $n = n' + \text{i}n''$, where both n' and n'' are positive. (It is assumed that $\mu = 1.0$ at optical frequencies.) The bottom of the dielectric is coated with a perfect conductor, as shown.



3 Pts a) Write expressions for the E- and H-fields of the incident plane-wave in the region $z \geq d$.

3 Pts b) Write expressions for the E- and H-fields of the reflected plane-wave in the region $z \geq d$.

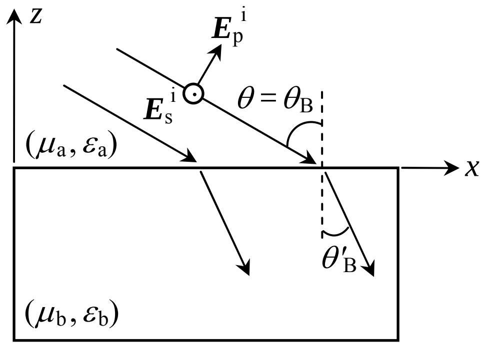
4 Pts c) Express the E- and H-fields inside the absorptive dielectric ($0 \leq z \leq d$) as a superposition of two plane-waves, one propagating downward, the other upward. (Your field profiles must satisfy the boundary conditions at $z = 0$.)

4 Pts d) Match the boundary conditions at $z = d$, and determine the amplitudes of the reflected and transmitted beams as functions of E_{xo}^i.

Continued on the reverse side …

4 Pts e) Use the Poynting vector at $z = d^-$ to determine the time-averaged rate of absorption of energy in the dielectric material.

Problem 3) A homogeneous plane-wave is incident at the interface between two *transparent* media at an oblique incidence angle θ. The incidence and transmittance media are specified by their parameters (μ_a, ε_a) and (μ_b, ε_b), respectively. For each medium, transparency dictates that μ and ε are real-valued, either both positive or both negative.



4 Pts a) Find an expression for the squared tangent of the Brewster angle, $\tan^2\theta_{Bp}$, in terms of $\mu_a, \varepsilon_a, \mu_b$, and ε_b. (**Note**: θ_{Bp} is the incidence angle at which ρ_p, the Fresnel reflection coefficient for p-polarized light, vanishes.)

4 Pts b) Repeat part (a) for s-polarized light by finding an expression for $\tan^2\theta_{Bs}$.

6 Pts c) Is it possible to have a set of parameters (μ_a, ε_a) and (μ_b, ε_b), as described above, for which two Brewster's angles exist, one for p-light and another for s-light? Explain.

Hint: The Fresnel reflection coefficients for p- and s-polarized light are given by

$$\rho_p = E^r_{xo}/E^i_{xo} = \frac{\varepsilon_a\sqrt{\mu_b\varepsilon_b - (ck_x/\omega)^2} - \varepsilon_b\sqrt{\mu_a\varepsilon_a - (ck_x/\omega)^2}}{\varepsilon_a\sqrt{\mu_b\varepsilon_b - (ck_x/\omega)^2} + \varepsilon_b\sqrt{\mu_a\varepsilon_a - (ck_x/\omega)^2}},$$

$$\rho_s = E^r_{yo}/E^i_{yo} = \frac{\mu_b\sqrt{\mu_a\varepsilon_a - (ck_x/\omega)^2} - \mu_a\sqrt{\mu_b\varepsilon_b - (ck_x/\omega)^2}}{\mu_b\sqrt{\mu_a\varepsilon_a - (ck_x/\omega)^2} + \mu_a\sqrt{\mu_b\varepsilon_b - (ck_x/\omega)^2}}.$$
