1) a) $$\hat{f}(t) = \frac{1}{2\pi}\int_{-\Omega}^{\Omega}\left\{\int_{t_1}^{t_2} f(t')e^{-i\omega t'}dt'\right\}e^{+i\omega t}d\omega =$$

$$\frac{1}{2\pi}\int_{t_1}^{t_2} f(t')\left\{\int_{-\Omega}^{\Omega} e^{i\omega(t-t')}d\omega\right\}dt' = \frac{1}{2\pi}\int_{t_1}^{t_2} f(t')\left.\frac{e^{i\omega(t-t')}}{i(t-t')}\right|_{-\Omega}^{\Omega}dt'$$

$$= \int_{t_1}^{t_2} f(t')\frac{e^{i\Omega(t-t')} - e^{-i\Omega(t-t')}}{2\pi i(t-t')}dt' = \int_{t_1}^{t_2} f(t')\frac{\sin[\Omega(t-t')]}{\pi(t-t')}dt'$$

$$= \int_{t_1}^{t_2} f(t')\left(\frac{\Omega}{\pi}\right)\frac{\sin[\Omega(t-t')]}{\Omega(t-t')}dt' \Rightarrow \hat{f}(t) = \int_{t_1}^{t_2} f(t')\left(\frac{\Omega}{\pi}\right)\operatorname{Sinc}\left[\frac{\Omega}{\pi}(t-t')\right]dt'$$

b) Sinc(t) is peaked at $t=0$, is symmetric, and has unit area. Therefore, $\left(\frac{\Omega}{\pi}\right)\operatorname{Sinc}\left(\frac{\Omega}{\pi}t\right)$ is tall, narrow, and has unit area, which means that, in the limit $\Omega \to \infty$, the function $\frac{\Omega}{\pi}\operatorname{Sinc}\left(\frac{\Omega}{\pi}t\right) \to \delta(t)$. We thus have, for sufficiently large Ω,

$$\hat{f}(t) \simeq \int_{t_1}^{t_2} f(t')\delta(t-t')dt' \xrightarrow[\text{Property of } \delta(\cdot)]{\text{using the sifting}} \hat{f}(t) \xrightarrow[\Omega\to\infty]{} f(t);\quad t_1 < t < t_2$$

c) When $t < t_1$ or $t > t_2$, the function $\left(\frac{\Omega}{\pi}\right)\operatorname{Sinc}\left[\frac{\Omega}{\pi}(t-t')\right]$ is peaked at $t' = t$, which is outside the interval (t_1, t_2). Within the (t_1, t_2) interval, therefore, the sinc function is zero yielding $\hat{f}(t) = 0$.

2) $$k_x = (\omega/c)\sqrt{\mu_a\epsilon_a}\,\sin\theta \Rightarrow (ck_x/\omega)^2 = \mu_a\epsilon_a\sin^2\theta$$

$$\rho_p = \frac{\epsilon_a\sqrt{\mu_b\epsilon_b - (ck_x/\omega)^2} - \epsilon_b\sqrt{\mu_a\epsilon_a - (ck_x/\omega)^2}}{\epsilon_a\sqrt{\mu_b\epsilon_b - (ck_x/\omega)^2} + \epsilon_b\sqrt{\mu_a\epsilon_a - (ck_x/\omega)^2}}$$ ← Plane-electromagnetic waves Eq. (17a)

$$\rho_s = \frac{\mu_b\sqrt{\mu_a\epsilon_a - (ck_x/\omega)^2} - \mu_a\sqrt{\mu_b\epsilon_b - (ck_x/\omega)^2}}{\mu_b\sqrt{\mu_a\epsilon_a - (ck_x/\omega)^2} + \mu_a\sqrt{\mu_b\epsilon_b - (ck_x/\omega)^2}}$$ ← Eq. (19a)

a) $n_a = n_b \Rightarrow \mu_a \epsilon_a = \mu_b \epsilon_b \Rightarrow \frac{\epsilon_a}{\epsilon_b} = \frac{\mu_b}{\mu_a}$

$$\Rightarrow \rho_p = \frac{\epsilon_a - \epsilon_b}{\epsilon_a + \epsilon_b} \quad \text{and} \quad \rho_s = \frac{\mu_b - \mu_a}{\mu_b + \mu_a} \quad ✓$$

However, $\rho_p = \dfrac{\epsilon_a/\epsilon_b - 1}{\epsilon_a/\epsilon_b + 1} = \dfrac{\frac{\mu_b}{\mu_a} - 1}{\frac{\mu_b}{\mu_a} + 1} = \dfrac{\mu_b - \mu_a}{\mu_b + \mu_a} = \rho_s$ ✓

Thus $\rho_p = \rho_s$ not only at normal incidence, but also at all angles of incidence $0 \le \theta \le 90°$. In fact, ρ_p and ρ_s are indepedent of θ and have the same values irrespective of the angle of incidece.

b) $\frac{\mu_a}{\epsilon_a} = \frac{\mu_b}{\epsilon_b} \Rightarrow \mu_a \epsilon_b = \mu_b \epsilon_a$

$$\Rightarrow \rho_p = \frac{\epsilon_a \sqrt{\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta} - \epsilon_b \sqrt{\mu_a \epsilon_a} \cos\theta}{\epsilon_a \sqrt{\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta} + \epsilon_b \sqrt{\mu_a \epsilon_a} \cos\theta} = 0 \Rightarrow$$

$$\epsilon_a \sqrt{\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta} = \epsilon_b \sqrt{\mu_a \epsilon_a} \cos\theta \Rightarrow \epsilon_a^2 (\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta) = \epsilon_b^2 \mu_a \epsilon_a \cos^2\theta$$

$$\Rightarrow \epsilon_a \mu_b \epsilon_b - \mu_a \epsilon_a^2 \sin^2\theta = \mu_a \epsilon_b^2 (1 - \sin^2\theta) \Rightarrow \epsilon_b (\epsilon_a \mu_b - \mu_a \epsilon_b) = \mu_a (\epsilon_a^2 - \epsilon_b^2) \sin^2\theta$$

Equality of impedances means that $\epsilon_a \mu_b = \mu_a \epsilon_b$ ad, therefore, the left-hand side of the above equation is zero. Consequently the right-hand side must be zero, which means that $\sin\theta = 0$. We conclude that only at normal incidence, when $\theta = 0$, it is possible to have $\rho_p = 0$.

$$\rho_s = 0 \Rightarrow \mu_b \sqrt{\mu_a \epsilon_a - \mu_a \epsilon_a \sin^2\theta} - \mu_a \sqrt{\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta} = 0 \Rightarrow$$

$$\mu_b^2 \mu_a \epsilon_a \cos^2\theta = \mu_a^2 (\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta) \Rightarrow \mu_b^2 \epsilon_a (1 - \sin^2\theta) = \mu_a \mu_b \epsilon_b - \mu_a^2 \epsilon_a \sin^2\theta$$

$$\Rightarrow \mu_b (\underbrace{\mu_b \epsilon_a - \mu_a \epsilon_b}_{0}) = \epsilon_a (\mu_b^2 - \mu_a^2) \sin^2\theta \Rightarrow \sin\theta = 0 \leftarrow$$ only at normal incidece is $\rho_s = 0$

3) a) $\vec{\nabla}\cdot\vec{D} = \rho_{free} \Rightarrow \vec{D}_\perp(\vec{r}_0,t)$ is continuous, that is, $\vec{D}_\perp(\vec{r}_0^{\,-},t) = \vec{D}_\perp(\vec{r}_0^{\,+},t)$ unless there is surface charge density $\sigma_{free}(\vec{r}_0,t)$ at the surface, in which case $D_\perp(\vec{r}_0^{\,+},t) - \vec{D}_\perp(\vec{r}_0^{\,-},t) = \sigma_{free}(\vec{r}_0,t)$

b) $\vec{\nabla}\times\vec{H} = \vec{J}_{free} + \frac{\partial\vec{D}}{\partial t} \Rightarrow \vec{H}_\parallel(\vec{r}_0,t)$ is continuous, that is, $\vec{H}_\parallel(\vec{r}_0^{\,-},t) = \vec{H}_\parallel(\vec{r}_0^{\,+},t)$

unless there is surface current density $\vec{J}_s(\vec{r}_0,t)$ at the surface, in which case $\vec{H}_\parallel(\vec{r}_0^{\,+},t) - \vec{H}_\parallel(\vec{r}_0^{\,-},t)$ is equal in magnitude and perpendicular in direction to $\vec{J}_s(\vec{r}_0,t)$. Note that $\vec{J}_s$ in general could have contributions from both $\vec{J}_{free}$ and $\vec{J}_{bound} = \partial\vec{P}(\vec{r}_0,t)/\partial t$.

c) $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t \Rightarrow \vec{E}_\parallel(\vec{r}_0,t)$ is continuous, that is, $\vec{E}_\parallel(\vec{r}_0^{\,-},t) = \vec{E}_\parallel(\vec{r}_0^{\,+},t)$ unless there is surface current density of magnetic monopoles, $\partial\vec{M}/\partial t$ at the surface, in which case $\vec{E}_\parallel(\vec{r}_0^{\,+},t) - \vec{E}_\parallel(\vec{r}_0^{\,-},t)$ is equal in magnitude and perpendicular in direction to the surface magnetic current density associated with $\partial\vec{M}(\vec{r}_0,t)/\partial t$.

d) $\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{B}_\perp(\vec{r}_0,t)$ is continuous, that is, $\vec{B}_\perp(\vec{r}_0^{\,-},t) = \vec{B}_\perp(\vec{r}_0^{\,+},t)$.

4) a) $k = (\omega/c)\sqrt{\mu(\omega)\epsilon(\omega)} \Rightarrow k_1 = (\omega_1/c)\sqrt{\mu(\omega_1)\epsilon(\omega_1)}$, $k_2 = (\omega_2/c)\sqrt{\mu(\omega_2)\epsilon(\omega_2)}$.

b) $\frac{\partial}{\partial t}\mathcal{E}_E(x,t) = \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \left[E_1\cos(k_1x-\omega_1t) + E_2\cos(k_2x-\omega_2t)\right]\frac{\partial}{\partial t}\left[\epsilon_0\epsilon_1E_1\cos(k_1x-\omega_1t)\right.$

$\left. + \epsilon_0\epsilon_2E_2\cos(k_2x-\omega_2t)\right] = \epsilon_0\epsilon_1\omega_1E_1^2\cos(k_1x-\omega_1t)\sin(k_1x-\omega_1t) +$

$\epsilon_0\epsilon_2\omega_2E_2^2\cos(k_2x-\omega_2t)\sin(k_2x-\omega_2t) + \epsilon_0\epsilon_1\omega_1E_1E_2\sin(k_1x-\omega_1t)\cos(k_2x-\omega_2t)$

$+ \epsilon_0\epsilon_2\omega_2E_1E_2\cos(k_1x-\omega_1t)\sin(k_2x-\omega_2t) \Rightarrow$

$$\frac{\partial}{\partial t}\mathcal{E}_E(x,t)=\frac{1}{2}\epsilon_0\epsilon_1\omega_1E_1^2\sin[2(k_1x-\omega_1t)]+\frac{1}{2}\epsilon_0\epsilon_2\omega_2E_2^2\sin[2(k_2x-\omega_2t)]$$

$$+\frac{1}{2}\epsilon_0\epsilon_1\omega_1E_1E_2\left\{\sin[(k_1+k_2)x-(\omega_1+\omega_2)t]+\sin[(k_1-k_2)x-(\omega_1-\omega_2)t]\right\}$$

$$+\frac{1}{2}\epsilon_0\epsilon_2\omega_2E_1E_2\left\{\sin[(k_1+k_2)x-(\omega_1+\omega_2)t]-\sin[(k_1-k_2)x-(\omega_1-\omega_2)t]\right\}\Rightarrow$$

$$\frac{\partial\mathcal{E}_E(x,t)}{\partial t}=\frac{1}{2}\epsilon_0\epsilon_1\omega_1E_1^2\sin[2(k_1x-\omega_1t)]+\frac{1}{2}\epsilon_0\epsilon_2\omega_2E_2^2\sin[2(k_2x-\omega_2t)]$$

$$+\frac{1}{2}\epsilon_0(\epsilon_1\omega_1+\epsilon_2\omega_2)E_1E_2\sin[(k_1+k_2)x-(\omega_1+\omega_2)t]+\frac{1}{2}\epsilon_0(\epsilon_1\omega_1-\epsilon_2\omega_2)E_1E_2\sin[(k_1-k_2)x-(\omega_1-\omega_2)t]$$

$$\Rightarrow\mathcal{E}_E(x,t)=C_0+\frac{1}{4}\epsilon_0\epsilon_1E_1^2\cos[2(k_1x-\omega_1t)]+\frac{1}{4}\epsilon_0\epsilon_2E_2^2\cos[2(k_2x-\omega_2t)]$$

$$+\frac{1}{2}\epsilon_0\frac{\epsilon_1\omega_1+\epsilon_2\omega_2}{\omega_1+\omega_2}E_1E_2\cos[(k_1+k_2)x-(\omega_1+\omega_2)t]$$

$$+\frac{1}{2}\epsilon_0\frac{\epsilon_1\omega_1-\epsilon_2\omega_2}{\omega_1-\omega_2}E_1E_2\cos[(k_1-k_2)x-(\omega_1-\omega_2)t].$$

c) $$\frac{1}{T}\int_{t-T}^{t}\mathcal{E}_E(x,t')\,dt'=C_0-\frac{1}{8T\omega_1}\epsilon_0\epsilon_1E_1^2\cancelto{0}{\sin[2(k_1x-\omega_1t)]\Big|_{t-T}^{t}}-\frac{1}{8T\omega_2}\epsilon_0\epsilon_2E_2^2\cancelto{0}{\sin[2(k_2x-\omega_2t)]\Big|_{t-T}^{t}}$$

$$-\frac{1}{2T}\epsilon_0\frac{\epsilon_1\omega_1+\epsilon_2\omega_2}{(\omega_1+\omega_2)^2}E_1E_2\sin[(k_1+k_2)x-(\omega_1+\omega_2)t]\Big|_{t-T}^{t}-\frac{\epsilon_0}{2T}\frac{\epsilon_1\omega_1-\epsilon_2\omega_2}{(\omega_1-\omega_2)^2}E_1E_2\sin[(k_1-k_2)x-(\omega_1-\omega_2)t]\Big|_{t-T}^{t}$$

Noting that $\omega_1=m\Delta\omega$, $\omega_2=(m+1)\Delta\omega$, and $\omega_1+\omega_2=(2m+1)\Delta\omega$, we simplify the above as follows:

$$\frac{1}{T}\int_{t-T}^{t}\mathcal{E}_E(x,t')\,dt'=C_0-\frac{\epsilon_0}{T}\frac{\epsilon(\omega_c)}{2\omega_c}E_1E_2\sin\left[\frac{2\omega_cn(\omega_c)}{c}x-2\omega_ct\right]$$

$$+\frac{\epsilon_0}{T}\frac{\frac{d}{d\omega}[\omega\epsilon(\omega)]_{\omega_c}}{\Delta\omega}E_1E_2\sin\left[\frac{\omega_1n(\omega_1)-\omega_2n(\omega_2)}{c}x-(\omega_1-\omega_2)t\right]\Rightarrow$$

$$\frac{1}{T}\int_{t-T}^{t}\mathcal{E}_E(x,t')\,dt'=C_0-\frac{\epsilon_0E_1E_2}{\pi}\left\{\cancelto{\simeq 0}{\frac{\epsilon(\omega_c)}{2m+1}\sin\left[\frac{2\omega_cn(\omega_c)}{c}x-2\omega_ct\right]}+\frac{d}{d\omega}[\omega\epsilon(\omega)]_{\omega_c}\sin\left[\frac{\Delta\omega}{c}n_g\left(x-\frac{ct}{n_g}\right)\right]\right\}$$

d) The first term can be neglected because $2m+1=2\omega_c/\Delta\omega$ is very large. The remaining term shows how the energy moves with the system. The velocity of energy is c/n_g, the group velocity.