Problem 1)

a) $\rho_{12} = (n_1 - n_2)/(n_1 + n_2)$, $\tau_{12} = 2n_1/(n_1 + n_2)$.

$\rho_{21} = (n_2 - n_1)/(n_2 + n_1) = -\rho_{12}$, $\tau_{21} = 2n_2/(n_2 + n_1)$.

$\rho_{23} = (n_2 - n_3)/(n_2 + n_3) = (n_2 - n - \mathrm{i}\kappa)/(n_2 + n + \mathrm{i}\kappa)$, $\tau_{23} = 2n_2/(n_2 + n_3)$.

b) Immediately beneath the entrance facet, $E_0^{(\mathrm{a})}$ receives a contribution from $E_0^{(\mathrm{i})}$ via the transmission coefficient τ_{12}. A second contribution comes from $E_0^{(\mathrm{b})}$ upon reflection at the upper dielectric surface (reflection coefficient $= \rho_{21}$). However, $E_0^{(\mathrm{b})}$ itself is obtained from $E_0^{(\mathrm{a})}$ after a downward propagation through the thickness d, reflection at the substrate interface (reflection coefficient $= \rho_{23}$), and an upward propagation, again through the thickness d. The self-consistency equation for $E_0^{(\mathrm{a})}$ may thus be written as follows:

$$E_0^{(\mathrm{a})} = \tau_{12} E_0^{(\mathrm{i})} + \rho_{21}\rho_{23} \exp(2\mathrm{i}n_2 k_0 d)\, E_0^{(\mathrm{a})} \quad \rightarrow \quad E_0^{(\mathrm{a})} = \frac{\tau_{12}}{1 - \rho_{21}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)} E_0^{(\mathrm{i})}.$$

c) The E-field amplitude transmitted into the substrate is obtained by propagating $E_0^{(\mathrm{a})}$ downward through the thickness d, then multiplying by τ_{23} to account for transmission from immediately above to immediately below the dielectric-substrate interface. We will have

$$E_0^{(\mathrm{t})} = \tau_{23} \exp(\mathrm{i}n_2 k_0 d)\, E_0^{(\mathrm{a})} = \frac{\tau_{12}\tau_{23}\exp(\mathrm{i}2\pi n_2 d/\lambda_0)}{1 - \rho_{21}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)} E_0^{(\mathrm{i})}.$$

d) The reflected E-field amplitude at the top of the dielectric layer has two contributions. The first comes from direct reflection from the top facet of the incident amplitude $E_0^{(\mathrm{i})}$. The second contribution comes from $E_0^{(\mathrm{b})}$ after multiplication by τ_{21}. However, $E_0^{(\mathrm{b})}$ itself arises from the propagation of $E_0^{(\mathrm{a})}$ downward through the thickness d, reflection at the substrate interface, then upward propagation through the thickness d of the dielectric layer. We will have

$$\begin{aligned} E_0^{(\mathrm{r})} &= \rho_{12} E_0^{(\mathrm{i})} + \tau_{21}\rho_{23} \exp(2\mathrm{i}n_2 k_0 d)\, E_0^{(\mathrm{a})} \\ &= \left[\rho_{12} + \frac{\tau_{12}\tau_{21}\rho_{23}\exp(2\mathrm{i}n_2 k_0 d)}{1 - \rho_{21}\rho_{23}\exp(2\mathrm{i}n_2 k_0 d)}\right] E_0^{(\mathrm{i})} = \left[\frac{\rho_{12} + (\tau_{12}\tau_{21} - \rho_{12}\rho_{21})\rho_{23}\exp(2\mathrm{i}n_2 k_0 d)}{1 - \rho_{21}\rho_{23}\exp(2\mathrm{i}n_2 k_0 d)}\right] E_0^{(\mathrm{i})}. \end{aligned}$$

Now, using the expressions for ρ_{12}, τ_{12}, ρ_{21}, τ_{21} obtained in part (a), we write

$$\tau_{12}\tau_{21} - \rho_{12}\rho_{21} = \frac{4n_1 n_2}{(n_1+n_2)^2} + \frac{(n_1-n_2)^2}{(n_1+n_2)^2} = 1.0$$

Consequently,

$$E_0^{(\mathrm{r})} = \frac{\rho_{12} + \rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)}{1 + \rho_{12}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)} E_0^{(\mathrm{i})}.$$

For a given refractive index n_2, the thickness d of the dielectric layer can be adjusted to control the reflectance of the bare substrate.

e) When $d = m\lambda_0/(2n_2)$, the phase-factor $\exp(\mathrm{i}4\pi n_2 d/\lambda_0)$ appearing in the preceding equation becomes equal to 1.0. We will then have

$$E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})} = \frac{\rho_{12}+\rho_{23}}{1+\rho_{12}\rho_{23}} = \frac{\left(\frac{n_1-n_2}{n_1+n_2}\right)+\left(\frac{n_2-n_3}{n_2+n_3}\right)}{1+\left(\frac{n_1-n_2}{n_1+n_2}\right)\left(\frac{n_2-n_3}{n_2+n_3}\right)} = \frac{(n_1-n_2)(n_2+n_3)+(n_2-n_3)(n_1+n_2)}{(n_1+n_2)(n_2+n_3)+(n_1-n_2)(n_2-n_3)}$$

$$= \frac{n_1n_2+n_1n_3-n_2^2-n_2n_3+n_1n_2+n_2^2-n_1n_3-n_2n_3}{n_1n_2+n_1n_3+n_2^2+n_2n_3+n_1n_2-n_1n_3-n_2^2+n_2n_3} = \frac{2n_1n_2-2n_2n_3}{2n_1n_2+2n_2n_3} = \frac{n_1-n_3}{n_1+n_3}.$$

Clearly, the overall reflection coefficient $E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})}$ in this case is independent of n_2, having the value it would have if the beam was directly incident from free space onto the substrate.

Problem 2) By definition $\rho_\mathrm{p} = E_{x0}^{(\mathrm{r})}/E_{x0}^{(\mathrm{i})}$. We shall also invoke the generalized form of Snell's law, $k_x^{(\mathrm{r})} = k_x^{(\mathrm{i})}$, and the dispersion relation $k_x^2 + k_z^2 = (\omega/c)^2 n_0^2(\omega)$.

a) $$\boldsymbol{k}^{(\mathrm{i})} = n_0(\omega)(\omega/c)(\sin\theta\,\hat{\boldsymbol{x}} - \cos\theta\,\hat{\boldsymbol{z}}).$$

$$\boldsymbol{E}_\mathrm{p}^{(\mathrm{i})}(\boldsymbol{r},t) = \left(E_{x0}^{(\mathrm{i})}\hat{\boldsymbol{x}} + E_{z0}^{(\mathrm{i})}\hat{\boldsymbol{z}}\right)\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{i})}\cdot\boldsymbol{r} - \omega t\right)\right].$$

From Maxwell's 1st equation: $\boldsymbol{\nabla}\cdot\boldsymbol{E} = 0 \quad\rightarrow\quad \boldsymbol{k}^{(\mathrm{i})}\cdot\boldsymbol{E}_\mathrm{p}^{(\mathrm{i})} = 0 \quad\rightarrow\quad E_{z0}^{(\mathrm{i})} = (\tan\theta)E_{x0}^{(\mathrm{i})}.$

From Maxwell's 3rd equation: $\boldsymbol{\nabla}\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \quad\rightarrow\quad \boldsymbol{k}^{(\mathrm{i})}\times\boldsymbol{E}_\mathrm{p}^{(\mathrm{i})} = \mu_0\omega\boldsymbol{H}_0^{(\mathrm{i})}$

$$\rightarrow\quad \boldsymbol{H}_0^{(\mathrm{i})} = Z_0^{-1}n_0(\omega)(\sin\theta\,\hat{\boldsymbol{x}} - \cos\theta\,\hat{\boldsymbol{z}})\times(E_{x0}^{(\mathrm{i})}\hat{\boldsymbol{x}} + E_{z0}^{(\mathrm{i})}\hat{\boldsymbol{z}})$$
$$= -Z_0^{-1}n_0(\omega)[\sin\theta\,E_{z0}^{(\mathrm{i})} + \cos\theta\,E_{x0}^{(\mathrm{i})}]\hat{\boldsymbol{y}}$$
$$= -Z_0^{-1}n_0(\omega)E_{x_0}^{(\mathrm{i})}\hat{\boldsymbol{y}}/\cos\theta$$
$$= -Z_0^{-1}n_0(\omega)E_\mathrm{p}^{(\mathrm{i})}\hat{\boldsymbol{y}}.$$

Consequently, $\boldsymbol{H}^{(\mathrm{i})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{i})}\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{i})}\cdot\boldsymbol{r} - \omega t\right)\right].$

Applying similar procedures to the reflected beam, we find

$$\boldsymbol{k}^{(\mathrm{r})} = n_0(\omega)(\omega/c)(\sin\theta\,\hat{\boldsymbol{x}} + \cos\theta\,\hat{\boldsymbol{z}}).$$

$$\boldsymbol{E}_\mathrm{p}^{(\mathrm{r})}(\boldsymbol{r},t) = \left(E_{x0}^{(\mathrm{r})}\hat{\boldsymbol{x}} + E_{z0}^{(\mathrm{r})}\hat{\boldsymbol{z}}\right)\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r} - \omega t\right)\right].$$

From Maxwell's 1st equation: $\boldsymbol{\nabla}\cdot\boldsymbol{E} = 0 \quad\rightarrow\quad \boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{E}_\mathrm{p}^{(\mathrm{r})} = 0 \quad\rightarrow\quad E_{z0}^{(\mathrm{r})} = -(\tan\theta)E_{x0}^{(\mathrm{r})}.$

From Maxwell's 3rd equation: $\boldsymbol{\nabla}\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \quad\rightarrow\quad \boldsymbol{k}^{(\mathrm{r})}\times\boldsymbol{E}_\mathrm{p}^{(\mathrm{r})} = \mu_0\omega\boldsymbol{H}_0^{(\mathrm{r})}$

$$\rightarrow\quad \boldsymbol{H}_0^{(\mathrm{r})} = Z_0^{-1}n_0(\omega)(\sin\theta\,\hat{\boldsymbol{x}} + \cos\theta\,\hat{\boldsymbol{z}})\times(E_{x0}^{(\mathrm{r})}\hat{\boldsymbol{x}} + E_{z0}^{(\mathrm{r})}\hat{\boldsymbol{z}})$$
$$= -Z_0^{-1}n_0(\omega)[\sin\theta\,E_{z0}^{(\mathrm{r})} - \cos\theta\,E_{x0}^{(\mathrm{r})}]\hat{\boldsymbol{y}}$$
$$= Z_0^{-1}n_0(\omega)E_{x_0}^{(\mathrm{r})}\hat{\boldsymbol{y}}/\cos\theta$$
$$= Z_0^{-1}n_0(\omega)\rho_\mathrm{p}E_{x_0}^{(\mathrm{i})}\hat{\boldsymbol{y}}/\cos\theta$$
$$= Z_0^{-1}n_0(\omega)\rho_\mathrm{p}E_\mathrm{p}^{(\mathrm{i})}\hat{\boldsymbol{y}}.$$

Consequently, $\boldsymbol{H}^{(\mathrm{r})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{r})}\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r} - \omega t\right)\right].$

b) $\langle \boldsymbol{S}^{(\mathrm{i})}(\boldsymbol{r},t)\rangle = ½\mathrm{Re}\{\boldsymbol{E}_\mathrm{p}^{(\mathrm{i})} \times \boldsymbol{H}_0^{*(\mathrm{i})}\} = -½ Z_0^{-1} n_0(\omega)\mathrm{Re}\{E_\mathrm{p}^{(\mathrm{i})}(\cos\theta\,\hat{\boldsymbol{x}} + \sin\theta\,\hat{\boldsymbol{z}}) \times E_\mathrm{p}^{*(\mathrm{i})}\hat{\boldsymbol{y}}\}$

$$= ½ Z_0^{-1} n_0(\omega)\left|E_\mathrm{p}^{(\mathrm{i})}\right|^2(\sin\theta\,\hat{\boldsymbol{x}} - \cos\theta\,\hat{\boldsymbol{z}})$$

$$= ½ Z_0^{-1} n_0(\omega)\left|E_\mathrm{p}^{(\mathrm{i})}\right|^2 \hat{\boldsymbol{k}}^{(\mathrm{i})}.$$

$\langle \boldsymbol{S}^{(\mathrm{r})}(\boldsymbol{r},t)\rangle = ½\mathrm{Re}\{\boldsymbol{E}_\mathrm{p}^{(\mathrm{r})} \times \boldsymbol{H}_0^{*(\mathrm{r})}\} = ½ Z_0^{-1} n_0(\omega)\mathrm{Re}\{\rho_\mathrm{p} E_\mathrm{p}^{(\mathrm{i})}(\cos\theta\,\hat{\boldsymbol{x}} - \sin\theta\,\hat{\boldsymbol{z}}) \times \rho_\mathrm{p}^* E_\mathrm{p}^{*(\mathrm{i})}\hat{\boldsymbol{y}}\}$

$$= ½ Z_0^{-1} n_0(\omega)\left|\rho_\mathrm{p} E_\mathrm{p}^{(\mathrm{i})}\right|^2(\sin\theta\,\hat{\boldsymbol{x}} + \cos\theta\,\hat{\boldsymbol{z}})$$

$$= ½ Z_0^{-1} n_0(\omega)|\rho_\mathrm{p}|^2\left|E_\mathrm{p}^{(\mathrm{i})}\right|^2 \hat{\boldsymbol{k}}^{(\mathrm{r})}.$$

The time-averaged Poynting vectors of the incident and reflected beams are seen to be along the corresponding directions of propagation. The rate of flow of energy of the reflected beam is that of the incident beam multiplied by $|\rho_\mathrm{p}|^2$. The phase φ_p of the Fresnel reflection coefficient does *not* affect the reflectance of optical energy at the interface between the two media.

Problem 3) a) For the transmitted beam, the continuity of k_x yields $k_x^{(\mathrm{t})} = k_x^{(\mathrm{i})} = (\omega/c)n_0 \sin\theta$. Also, the E-field amplitude immediately beneath the interface will be $\boldsymbol{E}_\mathrm{s}^{(\mathrm{t})} = \tau_\mathrm{s} E_\mathrm{s}^{(\mathrm{i})}\hat{\boldsymbol{y}}$. Thus,

$$\boldsymbol{k}^{(\mathrm{t})} = k_x\hat{\boldsymbol{x}} + k_z^{(\mathrm{t})}\hat{\boldsymbol{z}} = (\omega/c)\left[n_0\sin\theta\,\hat{\boldsymbol{x}} - \sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2\sin^2\theta}\,\hat{\boldsymbol{z}}\right].$$

$$\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t) = \tau_s E_s^{(\mathrm{i})}\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r} - \omega t\right)\right]\hat{\boldsymbol{y}}.$$

The square root must be chosen such that the imaginary part of $k_z^{(\mathrm{t})}$ is *negative*, so that the field amplitude will decay exponentially as $z \to -\infty$.

From Maxwell's 3rd equation: $\boldsymbol{\nabla}\times\boldsymbol{E} = -\partial\boldsymbol{B}/\partial t \quad \rightarrow \quad \boldsymbol{k}^{(\mathrm{t})} \times \tau_s \boldsymbol{E}_s^{(\mathrm{i})} = \mu_0\omega\boldsymbol{H}_0^{(\mathrm{t})}$

$$\rightarrow \quad \boldsymbol{H}_0^{(\mathrm{t})} = Z_0^{-1}\left[n_0\sin\theta\,\hat{\boldsymbol{x}} - \sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2\sin^2\theta}\,\hat{\boldsymbol{z}}\right] \times \tau_s E_s^{(\mathrm{i})}\hat{\boldsymbol{y}}$$

$$= Z_0^{-1}\tau_s E_\mathrm{s}^{(\mathrm{i})}\left[\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2\sin^2\theta}\,\hat{\boldsymbol{x}} + n_0\sin\theta\,\hat{\boldsymbol{z}}\right].$$

Consequently, $\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{t})}\exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r} - \omega t\right)\right].$

b) $\langle \boldsymbol{S}(\boldsymbol{r},t)\rangle = ½\mathrm{Re}\{\boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}^*(\boldsymbol{r},t)\}$

$$= ½\mathrm{Re}\left\{\tau_s E_s^{(\mathrm{i})}\exp[\mathrm{i}(k_x x + k_z^{(\mathrm{t})} z)]\,\hat{\boldsymbol{y}}\right.$$

$$\left.\times Z_0^{-1}\tau_s^* E_\mathrm{s}^{*(\mathrm{i})}\left[\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2\sin^2\theta}^{\,*}\,\hat{\boldsymbol{x}} + n_0\sin\theta\,\hat{\boldsymbol{z}}\right]\exp[-\mathrm{i}(k_x x + k_z^{*(\mathrm{t})} z)]\right\}$$

$$= ½ Z_0^{-1}\left|\tau_s E_\mathrm{s}^{(\mathrm{i})}\right|^2\left[n_0\sin\theta\,\hat{\boldsymbol{x}} - \mathrm{Re}\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2\sin^2\theta}\,\hat{\boldsymbol{z}}\right]\exp\{-2\mathrm{Im}[k_z^{(\mathrm{t})}]z\}.$$

As pointed out earlier, $\mathrm{Im}[k_z^{(\mathrm{t})}]$ is negative and, therefore, $\langle \boldsymbol{S}(\boldsymbol{r},t)\rangle$ decays exponentially as $z \to -\infty$.

Problem 4) The magnetization distribution $\boldsymbol{M}(\boldsymbol{r},t)$ does not produce any (bound) electrical charges. Therefore $\rho_{\text{bound}}^{(e)}(\boldsymbol{r},t)=0$. The absence of electrical charge implies that the scalar potential (in the Lorenz gauge) is also absent in this problem, that is, $\psi(\boldsymbol{r},t)=0$.

Since this is a magnetostatic problem (i.e., the magnetization is time-independent), the bound electric current-density $\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t)$ and, consequently, the vector potential $\boldsymbol{A}(\boldsymbol{r},t)$, will also be time-independent. As a result, we will have $\boldsymbol{E}(\boldsymbol{r},t)=-\boldsymbol{\nabla}\psi(\boldsymbol{r},t)-\partial\boldsymbol{A}(\boldsymbol{r},t)/\partial t=0$.

a)
$$\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t)=\mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r},t)=\mu_0^{-1}\boldsymbol{\nabla}\times[m_0\delta(x)\delta(y)\hat{\boldsymbol{z}}]$$
$$=\mu_0^{-1}m_0[\delta(x)\delta'(y)\hat{\boldsymbol{x}}-\delta'(x)\delta(y)\hat{\boldsymbol{y}}]$$

b) The symmetry of the problem allows us to choose the observation point $\boldsymbol{r}$ as an arbitrary point in the xy-plane, where $z=0$. In other words, $\boldsymbol{r}=x\hat{\boldsymbol{x}}+y\hat{\boldsymbol{y}}$. Also, since the current-density is time independent, the term $t-|\boldsymbol{r}-\boldsymbol{r}'|/c$ can be dropped from the vector potential formula. We will have

$$\boldsymbol{A}(\boldsymbol{r})=\frac{\mu_0}{4\pi}\iiint_{-\infty}^{\infty}\frac{\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}'=\frac{m_0}{4\pi}\iiint_{-\infty}^{\infty}\frac{\delta(x')\delta'(y')\hat{\boldsymbol{x}}-\delta'(x')\delta(y')\hat{\boldsymbol{y}}}{\sqrt{(x-x')^2+(y-y')^2+z'^2}}\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'$$

Sifting property of $\delta(x')$ and $\delta(y')$
$$=\frac{m_0}{4\pi}\left[\hat{\boldsymbol{x}}\iint_{-\infty}^{\infty}\frac{\delta'(y')}{\sqrt{x^2+(y-y')^2+z'^2}}\mathrm{d}y'\mathrm{d}z'-\hat{\boldsymbol{y}}\iint_{-\infty}^{\infty}\frac{\delta'(x')}{\sqrt{(x-x')^2+y^2+z'^2}}\mathrm{d}x'\mathrm{d}z'\right]$$

Sifting property of $\delta'(x')$ and $\delta'(y')$
$$=\frac{m_0}{4\pi}\left[-\hat{\boldsymbol{x}}\int_{-\infty}^{\infty}\frac{y}{(x^2+y^2+z'^2)^{3/2}}\mathrm{d}z'+\hat{\boldsymbol{y}}\int_{-\infty}^{\infty}\frac{x}{(x^2+y^2+z'^2)^{3/2}}\mathrm{d}z'\right]$$
$$=\frac{m_0}{2\pi}(-y\hat{\boldsymbol{x}}+x\hat{\boldsymbol{y}})\int_0^{\infty}\frac{\mathrm{d}z'}{(x^2+y^2+z'^2)^{3/2}}=\frac{m_0}{2\pi}(-y\hat{\boldsymbol{x}}+x\hat{\boldsymbol{y}})\left.\frac{z'}{(x^2+y^2)\sqrt{x^2+y^2+z'^2}}\right|_{z'=0}^{\infty}$$
$$=\frac{m_0}{2\pi}\left(\frac{x\hat{\boldsymbol{y}}-y\hat{\boldsymbol{x}}}{x^2+y^2}\right)=\left(\frac{m_0}{2\pi}\right)\frac{\hat{\boldsymbol{z}}\times(x\hat{\boldsymbol{x}}+y\hat{\boldsymbol{y}})}{x^2+y^2}=\left(\frac{m_0}{2\pi}\right)\frac{\hat{\boldsymbol{z}}\times\boldsymbol{r}}{r^2}=\frac{m_0\hat{\boldsymbol{\phi}}}{2\pi r}.$$
cylindrical coordinates

c)
$$\boldsymbol{B}(\boldsymbol{r},t)=\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r},t)=-\frac{\partial A_\phi}{\partial z}\hat{\boldsymbol{r}}+\frac{\partial(rA_\phi)}{r\partial r}\hat{\boldsymbol{z}}=\frac{\partial(m_0/2\pi)}{r\partial r}\hat{\boldsymbol{z}}=0.$$

The B-field, and also the H-field, are thus seen to be zero everywhere outside the wire—even though the vector potential is not zero. Note that on the z-axis itself, the curl of $\boldsymbol{A}(\boldsymbol{r})$ is *not* zero. Using the definition of Curl ($\boldsymbol{\nabla}\times$) as the integral of $\boldsymbol{A}(\boldsymbol{r})$ around a small loop, normalized by the loop area, the B-field inside the wire is readily found to be $m_0\delta(x)\delta(y)\hat{\boldsymbol{z}}$. This is simply the magnetization $\boldsymbol{M}(\boldsymbol{r})$ of the wire. Considering that $\boldsymbol{B}=\mu_0\boldsymbol{H}+\boldsymbol{M}$, we conclude that the H-field inside the wire is zero as well.