Problem 1)

a) $\boldsymbol{P}(\boldsymbol{r},t) = p_0\delta(x)\delta(y)\hat{\boldsymbol{z}} \quad \rightarrow \quad \rho_{\text{bound}}^{(\text{e})} = -\boldsymbol{\nabla}\cdot\boldsymbol{P} = -\frac{\partial P_z}{\partial z} = 0$ and $\boldsymbol{J}_{\text{bound}}^{(\text{e})} = \frac{\partial \boldsymbol{P}}{\partial t} = 0.$

b) Since the wire has no charge and no current, both its scalar and vector potentials must be zero.

c) In the absence of charge and current, there will be no electric and no magnetic fields.

d) $\boldsymbol{M}(\boldsymbol{r},t) = m_0\delta(x)\delta(y)\hat{\boldsymbol{z}} \quad \rightarrow \quad \boldsymbol{J}_{\text{bound}}^{(\text{e})} = \mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{M} = \mu_0^{-1}m_0[\delta(x)\delta'(y)\hat{\boldsymbol{x}} - \delta'(x)\delta(y)\hat{\boldsymbol{y}}].$

Since the wire has no electric charges, its scalar potential is zero, that is, $\psi(\boldsymbol{r},t) = 0$. As for vector potential, since the electric current is constant in time, the wire's vector potential will be time-independent. We thus write

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\iiint_{-\infty}^{\infty}\frac{\boldsymbol{J}_{\text{bound}}^{(\text{e})}(\tilde{\boldsymbol{r}})}{|\boldsymbol{r}-\tilde{\boldsymbol{r}}|}d\tilde{x}d\tilde{y}d\tilde{z} = \frac{m_0}{4\pi}\iiint_{-\infty}^{\infty}\frac{\delta(\tilde{x})\delta'(\tilde{y})\hat{\boldsymbol{x}} - \delta'(\tilde{x})\delta(\tilde{y})\hat{\boldsymbol{y}}}{\sqrt{(x-\tilde{x})^2+(y-\tilde{y})^2+(z-\tilde{z})^2}}d\tilde{x}d\tilde{y}d\tilde{z}$$

Use sifting properties of $\delta(\cdot)$ and $\delta'(\cdot)$. →

$$= \frac{m_0}{4\pi}\int_{-\infty}^{\infty}\frac{(-y\hat{\boldsymbol{x}}+x\hat{\boldsymbol{y}})}{[x^2+y^2+(z-\tilde{z})^2]^{3/2}}d\tilde{z} = \frac{m_0(-y\hat{\boldsymbol{x}}+x\hat{\boldsymbol{y}})}{4\pi(x^2+y^2)}\int_{-\infty}^{\infty}\frac{d\tilde{z}}{\sqrt{x^2+y^2}\,\{1+[(z-\tilde{z})/\sqrt{x^2+y^2}\,]^2\}^{3/2}}$$

Change variable to $\zeta = \frac{z-\tilde{z}}{\sqrt{x^2+y^2}}$. →

$$= \frac{m_0(-y\hat{\boldsymbol{x}}+x\hat{\boldsymbol{y}})}{4\pi(x^2+y^2)}\int_{-\infty}^{\infty}\frac{d\zeta}{(1+\zeta^2)^{3/2}} = \frac{m_0\hat{\boldsymbol{\phi}}}{4\pi\sqrt{x^2+y^2}}\left.\frac{\zeta}{\sqrt{1+\zeta^2}}\right|_{-\infty}^{\infty} = \frac{m_0\hat{\boldsymbol{\phi}}}{2\pi\rho}$$

← Switch to cylindrical coordinates (ρ,ϕ,z).

Considering that the scalar potential is zero and the vector potential is time-independent, the E-field surrounding the magnetic wire is found to be zero, that is, $\boldsymbol{E}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\psi - \partial\boldsymbol{A}/\partial t = 0$. As for the magnetic field, the curl of $\boldsymbol{A}(\boldsymbol{r})$ can be readily calculated in cylindrical coordinates and seen to be zero everywhere, except, along the z-axis, where $\boldsymbol{A}(\boldsymbol{r})$ is singular. Using the definition of the curl operator in the vicinity of the z-axis, we find that $\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{\nabla}\times\boldsymbol{A} = m_0\delta(x)\delta(y)\hat{\boldsymbol{z}}$. This, of course, is a consequence of the fact that, by definition, $\boldsymbol{B} = \mu_0\boldsymbol{H} + \boldsymbol{M}$, and that, in the absence of magnetic charges, i.e., $\rho_{\text{bound}}^{(\text{m})} = -\boldsymbol{\nabla}\cdot\boldsymbol{M} = 0$, the H-field everywhere is zero. Consequently, the B-field exists only within the wire, where $\boldsymbol{B} = \boldsymbol{M} = m_0\delta(x)\delta(y)\hat{\boldsymbol{z}}$.

Digression: An alternative means of calculating the magnetic wire's vector potential is the Fourier method, namely,

$$\boldsymbol{M}(\boldsymbol{k},\omega) = \int_{-\infty}^{\infty}\boldsymbol{M}(\boldsymbol{r},t)\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,d\boldsymbol{r}dt$$

$$= \int_{-\infty}^{\infty}m_0\delta(x)\delta(y)\hat{\boldsymbol{z}}\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,d\boldsymbol{r}dt = (2\pi)^2 m_0\delta(k_z)\delta(\omega)\hat{\boldsymbol{z}}.$$

$$\boldsymbol{J}_{\text{bound}}^{(\text{e})}(\boldsymbol{k},\omega) = \mathrm{i}\boldsymbol{k}\times\mu_0^{-1}\boldsymbol{M}(\boldsymbol{k},\omega) = (2\pi)^2\mu_0^{-1}m_0\delta(k_z)\delta(\omega)(\mathrm{i}\boldsymbol{k}\times\hat{\boldsymbol{z}}).$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty}\frac{\mu_0\boldsymbol{J}_{\text{bound}}^{(\text{e})}(\boldsymbol{k},\omega)}{k^2-(\omega/c)^2}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,d\boldsymbol{k}d\omega$$

$$= -\left(\frac{\mathrm{i}m_0}{4\pi^2}\right)\hat{\boldsymbol{z}}\times\int_{-\infty}^{\infty}\frac{\boldsymbol{k}\,\delta(k_z)\delta(\omega)}{k^2-(\omega/c)^2}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,d\boldsymbol{k}d\omega$$

$$= -\left(\frac{\mathrm{i}m_0}{4\pi^2}\right)\hat{\boldsymbol{z}}\times\int_{-\infty}^{\infty}\frac{\boldsymbol{k}_\parallel\exp(\mathrm{i}\boldsymbol{k}_\parallel\cdot\boldsymbol{\rho})}{k_\parallel^2}d\boldsymbol{k}_\parallel$$

← $\boldsymbol{k}_\parallel = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}}$, $\boldsymbol{\rho} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$

$$= -\left(\frac{\mathrm{i}m_0}{4\pi^2}\right)\hat{\boldsymbol{z}} \times \int_{k_\parallel=0}^{\infty}\int_{\varphi=0}^{2\pi} \frac{(k_\parallel \cos\varphi)\,\hat{\boldsymbol{\rho}}\,\exp(\mathrm{i}k_\parallel \rho\cos\varphi)}{k_\parallel^2} k_\parallel d\varphi dk_\parallel$$

$$= -\left(\frac{\mathrm{i}m_0}{4\pi^2}\right)(\hat{\boldsymbol{z}} \times \hat{\boldsymbol{\rho}}) \int_{k_\parallel=0}^{\infty} \left[\int_{\varphi=0}^{2\pi} \cos\varphi \exp(\mathrm{i}k_\parallel \rho\cos\varphi) d\varphi\right] dk_\parallel$$

$$= \frac{m_0\hat{\boldsymbol{\phi}}}{2\pi}\int_0^\infty J_1(k_\parallel \rho) dk_\parallel = \frac{m_0\hat{\boldsymbol{\phi}}}{2\pi\rho}.$$

← $J_1(\cdot)$ is Bessel function of first kind, 1st order.

This, of course, is the same solution for the vector potential as was obtained before.

Problem 2)

a) Dispersion relation: $k^2 = (\omega/c)^2\mu(\omega)\varepsilon(\omega) \quad\rightarrow\quad \boldsymbol{k} = \pm(\omega/c)\sqrt{\mu(\omega)\varepsilon(\omega)}\,\hat{\boldsymbol{k}}.$ (1)

In the above expression of $\boldsymbol{k}$, both plus and minus signs for the direction of propagation are retained. Here $\hat{\boldsymbol{k}}$ is an arbitrary unit vector, and the product $\mu(\omega)\varepsilon(\omega)$ is positive.

b) Faraday's law: $\boldsymbol{\nabla}\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad\rightarrow\quad \mathrm{i}\boldsymbol{k}\times\boldsymbol{E}_0 = \mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{H}_0 \quad\rightarrow\quad \boldsymbol{H}_0 = \frac{\boldsymbol{k}\times\boldsymbol{E}_0}{\omega\mu_0\mu(\omega)}.$ (2)

Considering that $\mu(\omega)$ appearing in the denominator in the above expression of $\boldsymbol{H}_0$ is negative, in what follows we will write it as $-\sqrt{\mu^2(\omega)}$. We will have

$$\boldsymbol{H}_0 = \pm\frac{(\omega/c)\sqrt{\mu(\omega)\varepsilon(\omega)}}{\omega\mu_0\mu(\omega)}\hat{\boldsymbol{k}}\times\boldsymbol{E}_0 = \mp\frac{\sqrt{\mu(\omega)\varepsilon(\omega)}}{c\mu_0\sqrt{\mu^2(\omega)}}\hat{\boldsymbol{k}}\times\boldsymbol{E}_0 = \mp\frac{\hat{\boldsymbol{k}}\times\boldsymbol{E}_0}{Z_0\sqrt{\mu(\omega)/\varepsilon(\omega)}}. \quad (3)$$

c) $\langle \boldsymbol{S}(\boldsymbol{r},t)\rangle = ½\mathrm{Re}[\boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}^*(\boldsymbol{r},t)] = ½\mathrm{Re}\,\{\boldsymbol{E}_0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\times\boldsymbol{H}_0^*\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\}$

$$= ½\mathrm{Re}(\boldsymbol{E}_0\times\boldsymbol{H}_0^*) = \mp\frac{\mathrm{Re}[\boldsymbol{E}_0\times(\hat{\boldsymbol{k}}\times\boldsymbol{E}_0^*)]}{2Z_0\sqrt{\mu(\omega)/\varepsilon(\omega)}} = \mp\frac{\mathrm{Re}[(\boldsymbol{E}_0\cdot\boldsymbol{E}_0^*)\hat{\boldsymbol{k}} - (\boldsymbol{E}_0\cdot\hat{\boldsymbol{k}})\boldsymbol{E}_0^*]}{2Z_0\sqrt{\mu(\omega)/\varepsilon(\omega)}}$$

$$= \mp\left[\frac{E_0'^2 + E_0''^2}{2Z_0\sqrt{\mu(\omega)/\varepsilon(\omega)}}\right]\hat{\boldsymbol{k}}. \quad (4)$$

($\boldsymbol{E}_0\cdot\hat{\boldsymbol{k}}$ → 0)

$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$

$\boldsymbol{E}_0\cdot\hat{\boldsymbol{k}} = 0$ because Maxwell's first equation, $\boldsymbol{\nabla}\cdot\boldsymbol{D} = 0$, yields $\mathrm{i}\boldsymbol{k}\cdot\varepsilon_0\varepsilon(\omega)\boldsymbol{E}_0 = 0$.

$\boldsymbol{E}_0\cdot\boldsymbol{E}_0^* = (\boldsymbol{E}_0' + \mathrm{i}\boldsymbol{E}_0'')\cdot(\boldsymbol{E}_0' - \mathrm{i}\boldsymbol{E}_0'') = E_0'^2 + E_0''^2$

Clearly, the choice of plus sign for $\boldsymbol{k}$ in Eq.(1) results in a minus sign for $\langle\boldsymbol{S}\rangle$ in Eq.(4), and vice-versa. The direction of energy flow is thus seen to be opposite that of the k-vector, the latter signifying the direction of phase propagation.

d) The Fresnel reflection coefficient from free space, where $\mu_a(\omega) = \varepsilon_a(\omega) = 1$, onto a negative-index medium having $\mu_b(\omega) = \varepsilon_b(\omega) = -1$, at normal incidence is the same for p- and s-polarized light, as follows:

$$\rho_p = \rho_s = \frac{\sqrt{\varepsilon_a/\mu_a} - \sqrt{\varepsilon_b/\mu_b}}{\sqrt{\varepsilon_a/\mu_a} + \sqrt{\varepsilon_b/\mu_b}} = \frac{1-1}{1+1} = 0. \quad (5)$$

The reflection coefficient is thus zero, because the negative-index medium is impedance matched to free space. The plane-wave transmitted into the negative-index medium must, therefore, have the same E-field and the same H-field as the incident wave, because of the required boundary conditions at the interface. From Eq.(3), we must now choose the plus sign for the H-field of the transmitted plane-wave into the negative-index medium, $\boldsymbol{H}_0 = \widehat{\boldsymbol{k}} \times \boldsymbol{E}_0/Z_0$. The choice of the plus sign should also be obvious from the necessity of having the transmitted beam carry energy away from the interface, that is, $\langle \boldsymbol{S} \rangle$ and $\widehat{\boldsymbol{k}}$ must be in the same direction. The choice of the plus sign for $\boldsymbol{H}_0$ then forces the k-vector in Eq.(1) to have the minus sign, that is, $\boldsymbol{k} = -(\omega/c)\widehat{\boldsymbol{k}}$. The transmitted plane-wave then has the following E- and H-fields:

$$\boldsymbol{E}^{(t)}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp\big[-\mathrm{i}(\omega/c)(\widehat{\boldsymbol{k}}\cdot\boldsymbol{r} + ct)\big], \tag{6a}$$

$$\boldsymbol{H}^{(t)}(\boldsymbol{r},t) = \big(\widehat{\boldsymbol{k}} \times \boldsymbol{E}_0/Z_0\big)\exp\big[-\mathrm{i}(\omega/c)(\widehat{\boldsymbol{k}}\cdot\boldsymbol{r} + ct)\big]. \tag{6b}$$

The phase of the E- and H-fields thus travels *toward* the interface with the speed of light c.

Problem 3)

a) Dispersion relation:

$$k^2 = k_x^2 + k_z^2 = (\omega/c)^2\mu_a(\omega)\varepsilon_a(\omega) \;\;\rightarrow\;\; k_z^{(\mathrm{i})} = \pm(\omega/c)\sqrt{\varepsilon_a(\omega) - (ck_x/\omega)^2}. \tag{1}$$

Since the incident wave is assumed to be evanescent, its k_z must be imaginary, and since it must decay away from the interface, only the plus sign will be acceptable. Therefore,

$$k_z^{(\mathrm{i})} = \mathrm{i}(\omega/c)\sqrt{(ck_x/\omega)^2 - \varepsilon_a(\omega)}. \tag{2}$$

Maxwell's first equation, $\boldsymbol{k}\cdot\boldsymbol{E}_0 = 0$, relates E_{z0} to E_{x0}, k_x, and k_z, as follows:

$$k_x E_{x0}^{(\mathrm{i})} + k_z^{(\mathrm{i})} E_{z0}^{(\mathrm{i})} = 0 \quad\rightarrow\quad E_{z0}^{(\mathrm{i})} = -k_x E_{x0}^{(\mathrm{i})}/k_z^{(\mathrm{i})}. \tag{3}$$

Maxwell's third equation, $\boldsymbol{k}\times\boldsymbol{E}_0 = \omega\mu_0\mu(\omega)\boldsymbol{H}_0$, now yields the magnetic field, namely,

$$\boldsymbol{H}_0^{(\mathrm{i})} = \frac{\big(k_x\widehat{\boldsymbol{x}} + k_z^{(\mathrm{i})}\widehat{\boldsymbol{z}}\big)\times\big(E_{x0}^{(\mathrm{i})}\widehat{\boldsymbol{x}} + E_{z0}^{(\mathrm{i})}\widehat{\boldsymbol{z}}\big)}{\mu_0\omega} = \frac{k_z^{(\mathrm{i})}E_{x0}^{(\mathrm{i})} - k_x E_{z0}^{(\mathrm{i})}}{\mu_0\omega}\widehat{\boldsymbol{y}} = \frac{k_x^2 + k_z^{(\mathrm{i})2}}{\mu_0\omega k_z^{(\mathrm{i})}}E_{x0}^{(\mathrm{i})}\widehat{\boldsymbol{y}}$$

$$= \frac{(\omega/c)^2\varepsilon_a(\omega)}{\mathrm{i}\mu_0(\omega^2/c)\sqrt{(ck_x/\omega)^2 - \varepsilon_a(\omega)}}E_{x0}^{(\mathrm{i})}\widehat{\boldsymbol{y}} = -\frac{\mathrm{i}\varepsilon_a(\omega)}{Z_0\sqrt{(ck_x/\omega)^2 - \varepsilon_a(\omega)}}E_{x0}^{(\mathrm{i})}\widehat{\boldsymbol{y}}. \tag{4}$$

Similar calculations for the transmitted plane-wave yield

$$k_z^{(\mathrm{t})} = -\mathrm{i}(\omega/c)\sqrt{(ck_x/\omega)^2 - \varepsilon_b(\omega)}. \tag{5}$$

$$\boldsymbol{H}_0^{(\mathrm{t})} = \frac{\mathrm{i}\varepsilon_b(\omega)}{Z_0\sqrt{(ck_x/\omega)^2 - \varepsilon_b(\omega)}}E_{x0}^{(\mathrm{t})}\widehat{\boldsymbol{y}}. \tag{6}$$

In the absence of a reflected wave, continuity of the tangential E- and H-fields at the boundary requires that $E_{x0}^{(\mathrm{t})} = E_{x0}^{(\mathrm{i})}$ and $H_{y0}^{(\mathrm{t})} = H_{y0}^{(\mathrm{i})}$. Therefore,

$$-\frac{\mathrm{i}\varepsilon_a(\omega)}{Z_0\sqrt{(ck_x/\omega)^2 - \varepsilon_a(\omega)}} = \frac{\mathrm{i}\varepsilon_b(\omega)}{Z_0\sqrt{(ck_x/\omega)^2 - \varepsilon_b(\omega)}} \quad\rightarrow\quad \frac{\varepsilon_a^2(\omega)}{(ck_x/\omega)^2 - \varepsilon_a(\omega)} = \frac{\varepsilon_b^2(\omega)}{(ck_x/\omega)^2 - \varepsilon_b(\omega)}$$

$$\rightarrow\;\; k_x = \left(\frac{\omega}{c}\right)\sqrt{\frac{\varepsilon_a(\omega)}{1 + [\varepsilon_a(\omega)/\varepsilon_b(\omega)]}}\,. \tag{7}$$

Note that the condition $-1 < \varepsilon_a(\omega)/\varepsilon_b(\omega) < 0$ ensures that $k_x > (\omega/c)\sqrt{\varepsilon_a(\omega)}$, which is necessary for the incident wave to be evanescent.

b) The time-averaged Poynting vector for the p-polarized plane-waves under consideration is given by

$$\begin{aligned}\langle \boldsymbol{S}(\boldsymbol{r},t)\rangle &= \tfrac{1}{2}\mathrm{Re}\{\boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)] \times \boldsymbol{H}_0^* \exp[-\mathrm{i}(\boldsymbol{k}^*\cdot\boldsymbol{r}-\omega t)]\}\\ &= \tfrac{1}{2}\exp(-2\boldsymbol{k}''\cdot\boldsymbol{r})\,\mathrm{Re}\big[(E_{x0}\hat{\boldsymbol{x}}+E_{z0}\hat{\boldsymbol{z}})\times H_{y0}^*\hat{\boldsymbol{y}}\big]\\ &= \tfrac{1}{2}\exp(-2\boldsymbol{k}''\cdot\boldsymbol{r})\,\mathrm{Re}\big(E_{x0}H_{y0}^*\hat{\boldsymbol{z}}-E_{z0}H_{y0}^*\hat{\boldsymbol{x}}\big)\\ &= \tfrac{1}{2}\exp(-2\boldsymbol{k}''\cdot\boldsymbol{r})\,\mathrm{Re}\big[(k_x/k_z)E_{x0}H_{y0}^*\big]\hat{\boldsymbol{x}}. \qquad (8)\end{aligned}$$

(Since $E_{x0}H_{y0}^*$ is imaginary, its real part vanishes.)

For the incident wave, we have

$$\langle S_x^{(\mathrm{i})}\rangle = \frac{(ck_x/\omega)\varepsilon_a(\omega)\exp\big[-2(\omega/c)\sqrt{(ck_x/\omega)^2-\varepsilon_a(\omega)}\,z\big]}{2Z_0[(ck_x/\omega)^2-\varepsilon_a(\omega)]}|E_{x0}^{(\mathrm{i})}|^2. \qquad (9)$$

Similarly, for the transmitted wave,

$$\langle S_x^{(\mathrm{t})}\rangle = \frac{(ck_x/\omega)\varepsilon_b(\omega)\exp\big[2(\omega/c)\sqrt{(ck_x/\omega)^2-\varepsilon_b(\omega)}\,z\big]}{2Z_0[(ck_x/\omega)^2-\varepsilon_b(\omega)]}|E_{x0}^{(\mathrm{t})}|^2. \qquad (10)$$

Note that the energy flow direction in the dielectric is opposite to that in the metallic medium.

c) In the case of and s-polarized incident wave, we will have

$$\boldsymbol{H}_0^{(\mathrm{i})} = \frac{(k_x\hat{\boldsymbol{x}}+k_z^{(\mathrm{i})}\hat{\boldsymbol{z}})\times E_{y0}^{(\mathrm{i})}\hat{\boldsymbol{y}}}{\mu_0\omega} = (E_{y0}^{(\mathrm{i})}/Z_0)\Big[(ck_x/\omega)\hat{\boldsymbol{z}}-\mathrm{i}\sqrt{(ck_x/\omega)^2-\varepsilon_a(\omega)}\,\hat{\boldsymbol{x}}\Big]. \qquad (11)$$

$$\boldsymbol{H}_0^{(\mathrm{t})} = \frac{(k_x\hat{\boldsymbol{x}}+k_z^{(\mathrm{t})}\hat{\boldsymbol{z}})\times E_{y0}^{(\mathrm{t})}\hat{\boldsymbol{y}}}{\mu_0\omega} = (E_{y0}^{(\mathrm{t})}/Z_0)\Big[(ck_x/\omega)\hat{\boldsymbol{z}}+\mathrm{i}\sqrt{(ck_x/\omega)^2-\varepsilon_b(\omega)}\,\hat{\boldsymbol{x}}\Big]. \qquad (12)$$

Clearly, the tangential components of both the E-field and the H-field cannot be continuous at the interface, because, as seen in Eqs.(11) and (12), $H_{x0}^{(\mathrm{i})} \neq H_{x0}^{(\mathrm{t})}$ when $E_{y0}^{(\mathrm{i})} = E_{y0}^{(\mathrm{t})}$.