Problem 7.73) In the following analysis, the polarization of the incident, reflected, and transmitted beams is taken to be along the x-axis, the speed of light in vacuum is denoted by c, and the impedance of free space is Z_0. The numerical value of Z_0 is $\sim 377\Omega$.

a)
$$\boldsymbol{E}^{(\mathrm{i})}(\boldsymbol{r},t) = E_0^{(\mathrm{i})}\hat{\boldsymbol{x}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{i})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{H}^{(\mathrm{i})}(\boldsymbol{r},t) = H_0^{(\mathrm{i})}\hat{\boldsymbol{y}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{i})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{k}^{(\mathrm{i})} = -(\omega/c)\hat{\boldsymbol{z}}; \qquad H_0^{(\mathrm{i})} = -E_0^{(\mathrm{i})}/Z_0.$$

$$\boldsymbol{E}^{(\mathrm{r})}(\boldsymbol{r},t) = E_0^{(\mathrm{r})}\hat{\boldsymbol{x}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{H}^{(\mathrm{r})}(\boldsymbol{r},t) = H_0^{(\mathrm{r})}\hat{\boldsymbol{y}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{r})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{k}^{(\mathrm{r})} = +(\omega/c)\hat{\boldsymbol{z}}; \qquad H_0^{(\mathrm{r})} = +E_0^{(\mathrm{r})}/Z_0.$$

$$\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t) = E_0^{(\mathrm{t})}\hat{\boldsymbol{x}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t) = H_0^{(\mathrm{t})}\hat{\boldsymbol{y}}\exp\big[\mathrm{i}\big(\boldsymbol{k}^{(\mathrm{t})}\cdot\boldsymbol{r}-\omega t\big)\big],$$
$$\boldsymbol{k}^{(\mathrm{t})} = -(n\omega/c)\hat{\boldsymbol{z}}; \qquad H_0^{(\mathrm{t})} = -nE_0^{(\mathrm{t})}/Z_0.$$

b) At normal incidence, the Fresnel reflection and transmission coefficients from vacuum (where $n_0 = 1$) to water (where $n_1 = 1.33$) are given by

$$\rho = \frac{n_0-n_1}{n_0+n_1} = -0.14163; \qquad \tau = \frac{2n_0}{n_0+n_1} = 0.85837.$$

Consequently, $E_0^{(\mathrm{r})} = -0.14163E_0^{(\mathrm{i})}$, and $E_0^{(\mathrm{t})} = 0.85837E_0^{(\mathrm{i})}$.

c) The energy flux per unit area per unit time is the time-averaged Poynting vector, that is,

$$\langle\boldsymbol{S}\rangle = ½\mathrm{Re}(\boldsymbol{E}\times\boldsymbol{H}^*) = ½\mathrm{Re}(E_0\hat{\boldsymbol{x}}\times H_0^*\hat{\boldsymbol{y}}) = \pm ½\, n|E_0|^2\hat{\boldsymbol{z}}/Z_0.$$

Thus for the incident beam

$$S_z^{(\mathrm{i})} = -½\left|E_0^{(\mathrm{i})}\right|^2/Z_0,$$

for the reflected beam

$$S_z^{(\mathrm{r})} = +½\left|E_0^{(\mathrm{r})}\right|^2/Z_0 = ½\,(-0.14163)^2\left|E_0^{(\mathrm{i})}\right|^2/Z_0 = ½\,(0.02006)\left|E_0^{(\mathrm{i})}\right|^2/Z_0,$$

and for the transmitted beam

$$S_z^{(\mathrm{t})} = -½\,(1.33)(0.85837)^2\left|E_0^{(\mathrm{i})}\right|^2/Z_0 = -½(0.97994)\left|E_0^{(\mathrm{i})}\right|^2/Z_0.$$

d) Since $0.97994 + 0.02006 = 1.0$, we conclude that the flux of incident energy is equal to the sum of the reflected and transmitted fluxes of energy. Therefore, energy is being conserved.