**Problem 7.71)**

a) $\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)+\frac{\partial\boldsymbol{D}(\boldsymbol{r},t)}{\partial t} \;\rightarrow\; -\left(\frac{\partial H_y}{\partial z}\right)\hat{\boldsymbol{x}}+\left(\frac{\partial H_y}{\partial x}\right)\hat{\boldsymbol{z}}=\varepsilon_0\frac{\partial\boldsymbol{E}(\boldsymbol{r},t)}{\partial t}$

$$\rightarrow\; -H_0k_z\cos(k_xx)\cos(k_zz-\omega_0t)\hat{\boldsymbol{x}}-H_0k_x\sin(k_xx)\sin(k_zz-\omega_0t)\hat{\boldsymbol{z}}=\varepsilon_0\frac{\partial\boldsymbol{E}(\boldsymbol{r},t)}{\partial t}$$

$$\rightarrow\; \boldsymbol{E}(\boldsymbol{r},t)=\frac{H_0k_z}{\varepsilon_0\omega_0}\cos(k_xx)\sin(k_zz-\omega_0t)\,\hat{\boldsymbol{x}}-\frac{H_0k_x}{\varepsilon_0\omega_0}\sin(k_xx)\cos(k_zz-\omega_0t)\,\hat{\boldsymbol{z}}.$$

b) $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\frac{\partial\boldsymbol{B}(\boldsymbol{r},t)}{\partial t} \;\rightarrow\; \left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)\hat{\boldsymbol{y}}=-\mu_0\frac{\partial\boldsymbol{H}}{\partial t}$

$$\rightarrow\left(\frac{H_0k_z^2}{\varepsilon_0\omega_0}\right)\cos(k_xx)\cos(k_zz-\omega_0t)+\left(\frac{H_0k_x^2}{\varepsilon_0\omega_0}\right)\cos(k_xx)\cos(k_zz-\omega_0t)$$
$$=\mu_0H_0\,\omega_0\cos(k_xx)\cos(k_zz-\omega_0t)$$

$$\rightarrow\; k_x^2+k_z^2=\mu_0\varepsilon_0\omega_0^2 \;\rightarrow\; k_x^2+k_z^2=(\omega_0/c)^2.$$

To ensure that both $k_x$ and $k_z$ are real-valued, we may define an angle $\theta$ (see the figure) such that $k_x=(\omega_0/c)\sin\theta$ and $k_z=(\omega_0/c)\cos\theta$.

c) The tangential component of the $E$-field must vanish at the inner surfaces of the perfect conductors. Therefore, the necessary and sufficient condition for the admissibility of the guided mode is $E_z(x=\pm\tfrac{1}{2}d,y,z,t)=0$, which is equivalent to $\sin(\pm\tfrac{1}{2}k_xd)=0$. We must thus have $\tfrac{1}{2}k_xd=m\pi$, where $m$ is an arbitrary integer. In other words, $k_x=(\omega_0/c)\sin\theta=2\pi m/d$, or, equivalently, $\sin\theta=m\lambda_0/d$.

d) The surface-charge-density is equal to $D_\perp$ in the immediate vicinity of the surface, that is, $|\sigma_s|=D_x=\varepsilon_0E_x$. The sign of $\sigma_s$ is positive if $D_\perp$ exits from the surface, and negative if $D_\perp$ enters into the surface. We have

$$\sigma_s(x=\pm\tfrac{1}{2}d,y,z,t)=\mp(H_0k_z/\omega_0)\cos(\tfrac{1}{2}k_xd)\sin(k_zz-\omega_0t)$$
$$=\mp(-1)^m(H_0k_z/\omega_0)\sin(k_zz-\omega_0t).$$

The surface-current-density is equal in magnitude and perpendicular in direction to $\boldsymbol{H}_\parallel$ in the immediate vicinity of the surface, that is, $|\boldsymbol{J}_s|=H_y$. The direction of the current is related to the direction of the magnetic field via the right-hand rule. We have

$$\boldsymbol{J}_s(x=\pm\tfrac{1}{2}d,y,z,t)=\mp H_0\hat{\boldsymbol{z}}\cos(\tfrac{1}{2}k_xd)\sin(k_zz-\omega_0t)=\mp(-1)^mH_0\hat{\boldsymbol{z}}\sin(k_zz-\omega_0t).$$

e) The charge-current continuity equation at the inner surfaces of the conductors may now be written as follows:

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}_s+\frac{\partial\sigma_s}{\partial t}=\frac{\partial J_{sz}}{\partial z}+\frac{\partial\sigma_s}{\partial t}$$
$$=\mp(-1)^m\,H_0\,k_z\cos(k_zz-\omega_0t)\pm(-1)^mH_0k_z\cos(k_zz-\omega_0t)=0.$$