Problem 7.70)

a) At the interface of the perfect conductor with the dielectric layer, the tangential E-field must vanish. Therefore, $E_x(x,y,z=0,t)=E_1\sin(\varphi_1)\cos(\omega_0 t)=0$, which leads to $\varphi_1=0$ or π. In what follows, we shall set $\varphi_1=0$.

b) $$\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\frac{\partial\boldsymbol{B}(\boldsymbol{r},t)}{\partial t}\quad\rightarrow\quad\left(\frac{\partial E_x}{\partial z}\right)\hat{\boldsymbol{y}}=-\mu_o\mu(\omega_0)\frac{\partial\boldsymbol{H}}{\partial t}$$

$$\rightarrow E_1k_1\cos(k_1z)\cos(\omega_0 t)=-\mu_o\frac{\partial H_y}{\partial t}\quad\rightarrow\quad\boldsymbol{H}(\boldsymbol{r},t)=-\left(\frac{E_1k_1}{\mu_o\omega_0}\right)\hat{\boldsymbol{y}}\cos(k_1z)\sin(\omega_0 t).$$

c) $$\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)+\frac{\partial\boldsymbol{D}(\boldsymbol{r},t)}{\partial t}\quad\rightarrow\quad-\left(\frac{\partial H_y}{\partial z}\right)\hat{\boldsymbol{x}}=\varepsilon_o\varepsilon(\omega_0)\frac{\partial\boldsymbol{E}(\boldsymbol{r},t)}{\partial t}$$

$$\rightarrow\quad-\left(\frac{E_1k_1^2}{\mu_o\omega_0}\right)\sin(k_1z)\sin(\omega_0 t)=-\varepsilon_o\varepsilon(\omega_0)\omega_0E_1\sin(k_1z)\sin(\omega_0 t)$$

$$\rightarrow\quad k_1^2=\mu_o\varepsilon_o\varepsilon(\omega_0)\omega_0^2\quad\rightarrow\quad k_1=(\omega_0/c)n(\omega_0).$$

d) Considering that $\rho_{\text{free}}=0$ and $\boldsymbol{D}=\varepsilon_o\varepsilon\boldsymbol{E}$, the first Maxwell equation becomes $\boldsymbol{\nabla}\cdot\boldsymbol{E}=0$. We thus have $\boldsymbol{\nabla}\cdot\boldsymbol{E}=\partial E_x/\partial x=0$. Similarly, since $\boldsymbol{B}=\mu_o\mu\boldsymbol{H}$, Maxwell's fourth equation becomes $\boldsymbol{\nabla}\cdot\boldsymbol{H}=0$, which is automatically satisfied given that $\boldsymbol{\nabla}\cdot\boldsymbol{H}=\partial H_y/\partial y=0$.

e) $$\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t)=-\frac{\partial\boldsymbol{B}(\boldsymbol{r},t)}{\partial t}\quad\rightarrow\quad\left(\frac{\partial E_x}{\partial z}\right)\hat{\boldsymbol{y}}=-\mu_o\frac{\partial\boldsymbol{H}}{\partial t}$$

$$\rightarrow E_0k_0\cos(k_0z+\varphi_0)\cos(\omega_0 t)=-\mu_o\frac{\partial H_y}{\partial t}$$

$$\rightarrow\quad\boldsymbol{H}(\boldsymbol{r},t)=-\left(\frac{E_0k_0}{\mu_o\omega_0}\right)\hat{\boldsymbol{y}}\cos(k_0z+\varphi_0)\sin(\omega_0 t).$$

Substitution for $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$ in Maxwell's second equation now yields

$$\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)+\frac{\partial\boldsymbol{D}(\boldsymbol{r},t)}{\partial t}\quad\rightarrow\quad-\left(\frac{\partial H_y}{\partial z}\right)\hat{\boldsymbol{x}}=\varepsilon_o\frac{\partial\boldsymbol{E}(\boldsymbol{r},t)}{\partial t}$$

$$\rightarrow\quad-\left(\frac{E_0k_0^2}{\mu_o\omega_0}\right)\sin(k_0z+\varphi_0)\sin(\omega_0 t)=-\varepsilon_o\omega_0E_0\sin(k_0z+\varphi_0)\sin(\omega_0 t)$$

$$\rightarrow\quad k_0^2=\mu_o\varepsilon_o\omega_0^2\quad\rightarrow\quad k_0=\omega_0/c.$$

f) At the interface located at $z=d$, both E_x and H_y must be continuous. We thus have

$$\begin{cases}E_1\sin(k_1d)\cos(\omega_0 t)=E_0\sin(k_0d+\varphi_0)\cos(\omega_0 t)\\-\left(\frac{E_1k_1}{\mu_o\omega_0}\right)\cos(k_1d)\sin(\omega_0 t)=-\left(\frac{E_0k_0}{\mu_o\omega_0}\right)\cos(k_0d+\varphi_0)\sin(\omega_0 t)\end{cases}$$

$$\rightarrow\quad\begin{cases}E_1\sin(k_1d)=E_0\sin(k_0d+\varphi_0)\\E_1k_1\cos(k_1d)=E_0k_0\cos(k_0d+\varphi_0)\end{cases}\quad\rightarrow\quad\begin{cases}\tan(k_1d)=n(\omega_0)\tan(k_0d+\varphi_0)\\E_1/E_0=\sin(k_0d+\varphi_0)\,/\sin(k_1d).\end{cases}$$

The above equations may now be solved for the values of φ_0 and E_1/E_0.