Problem 7.69)

a)
$$\boldsymbol{E}_1(\boldsymbol{r},t)=E_o\cos[n(\omega_1)(\omega_1/c)z-\omega_1 t]\hat{\boldsymbol{x}},\tag{1a}$$

$$\boldsymbol{H}_1(\boldsymbol{r},t)=n(\omega_1)Z_o^{-1}E_o\cos[n(\omega_1)(\omega_1/c)z-\omega_1 t]\hat{\boldsymbol{y}}.\tag{1b}$$

Similarly,

$$\boldsymbol{E}_2(\boldsymbol{r},t)=E_o\cos[n(\omega_2)(\omega_2/c)z-\omega_2 t]\hat{\boldsymbol{x}},\tag{2a}$$

$$\boldsymbol{H}_2(\boldsymbol{r},t)=n(\omega_2)Z_o^{-1}E_o\cos[n(\omega_2)(\omega_2/c)z-\omega_2 t]\hat{\boldsymbol{y}}.\tag{2b}$$

Here $c=1/\sqrt{\mu_o\varepsilon_o}$ is the speed of light in vacuum, while $Z_o=\sqrt{\mu_o/\varepsilon_o}$ is the impedance of free space.

b) The rate of flow of electromagnetic (EM) energy is given by the Poynting vector, as follows:

$$\begin{aligned}\boldsymbol{S}(\boldsymbol{r},t)=\boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t)&=E_o\{\cos[n(\omega_1)(\omega_1/c)z-\omega_1 t]+\cos[n(\omega_2)(\omega_2/c)z-\omega_2 t]\}\,\hat{\boldsymbol{x}}\\
&\quad\times Z_o^{-1}E_o\{n(\omega_1)\cos[n(\omega_1)(\omega_1/c)z-\omega_1 t]+n(\omega_2)\cos[n(\omega_2)(\omega_2/c)z-\omega_2 t]\}\,\hat{\boldsymbol{y}}\\
&=Z_o^{-1}E_o^2\{n(\omega_1)\cos^2[n(\omega_1)(\omega_1/c)z-\omega_1 t]+n(\omega_2)\cos^2[n(\omega_2)(\omega_2/c)z-\omega_2 t]\\
&\quad+[n(\omega_1)+n(\omega_2)]\cos[n(\omega_1)(\omega_1/c)z-\omega_1 t]\cos[n(\omega_2)(\omega_2/c)z-\omega_2 t]\}\hat{\boldsymbol{z}}\\
&=\tfrac{1}{2}Z_o^{-1}E_o^2\{[n(\omega_1)+n(\omega_2)]+n(\omega_1)\cos[2\omega_1 n(\omega_1)(z/c)-2\omega_1 t]+n(\omega_2)\cos[2\omega_2 n(\omega_2)(z/c)-2\omega_2 t]\\
&\quad+[n(\omega_1)+n(\omega_2)]\cos\{[\omega_1 n(\omega_1)+\omega_2 n(\omega_2)](z/c)-(\omega_1+\omega_2)t\}\\
&\quad+[n(\omega_1)+n(\omega_2)]\cos\{[\omega_1 n(\omega_1)-\omega_2 n(\omega_2)](z/c)-(\omega_1-\omega_2)t\}\}\,\hat{\boldsymbol{z}}.\end{aligned}\tag{3}$$

c) In the preceding expression, the terms with frequencies $2\omega_1$, $2\omega_2$, and $(\omega_1+\omega_2)$ are rapidly-oscillating functions of time which quickly average to zero. The first term, however, is a constant, and the last term, which varies slowly with time, co-propagates with the envelope of the beat signal. Dropping the rapidly-oscillating terms, we will have

$$\begin{aligned}S_z(\boldsymbol{r},t)&\approx\tfrac{1}{2}[n(\omega_1)+n(\omega_2)]Z_o^{-1}E_o^2\{1+\cos\{[\omega_2 n(\omega_2)-\omega_1 n(\omega_1)](z/c)-(\omega_2-\omega_1)t\}\}\\
&\approx[n(\omega_1)+n(\omega_2)]Z_o^{-1}E_o^2\cos^2\left\{\frac{1}{2}\Delta\omega\left(\frac{\mathrm{d}[\omega n(\omega)]}{c\,\mathrm{d}\omega}\bigg|_{\omega_o}z-t\right)\right\}.\end{aligned}\tag{4}$$

In the above equation, the rate-of-flow of the beat signal's EM energy is seen to travel along the z-axis at the constant velocity c/n_g, where $n_g=\mathrm{d}[\omega n(\omega)]/\mathrm{d}\omega|_{\omega=\omega_o}$ is the group refractive index of the medium at the center frequency ω_o of the beat signal. The energy flow-rate is thus seen to propagate along the z-axis at the group velocity $V_g=c/n_g$. Note that the final expression obtained in Eq.(4) is positive everywhere, whereas the rapidly-oscillating terms that were dropped from Eq.(3) keep switching direction (between $+z$ and $-z$) at very high frequencies.