Problem 7.67)

a) Continuity of E_x at the entrance facet: $E_x^{(\mathrm{i})} = E_x^{(1)} + E_x^{(2)}$.

Continuity of H_y at the entrance facet: $H_y^{(\mathrm{i})} = H_y^{(1)} + H_y^{(2)} \;\rightarrow\; Z_o^{-1} E_x^{(\mathrm{i})} = n Z_o^{-1} E_x^{(1)} - n Z_o^{-1} E_x^{(2)}$.

We can now solve the above equations for $E_x^{(1)}$ and $E_x^{(2)}$, as follows:

$$E_x^{(1)} = [(n+1)/2n] E_x^{(\mathrm{i})},$$

$$E_x^{(2)} = [(n-1)/2n] E_x^{(\mathrm{i})}.$$

The H-fields are subsequently found to be

$$H_y^{(1)} = [(n+1)/2Z_o] E_x^{(\mathrm{i})},$$

$$H_y^{(2)} = -[(n-1)/2Z_o] E_x^{(\mathrm{i})}.$$

The transmitted beam is obtained by matching the boundary conditions at the exit facet of the slab, and using the fact that the magnitude of the k-vector inside the slab is $nk_o = 2\pi n/\lambda_o$. We will have

$$E_x^{(\mathrm{t})} = E_x^{(1)} \exp(\mathrm{i} n k_o d) + E_x^{(2)} \exp(-\mathrm{i} n k_o d) = E_x^{(1)} \exp(\mathrm{i}\pi) + E_x^{(2)} \exp(-\mathrm{i}\pi) = -E_x^{(1)} - E_x^{(2)} = -E_x^{(\mathrm{i})},$$

$$H_y^{(\mathrm{t})} = H_y^{(1)} \exp(\mathrm{i} n k_o d) + H_y^{(2)} \exp(-\mathrm{i} n k_o d) = -H_y^{(1)} - H_y^{(2)} = -E_x^{(\mathrm{i})}/Z_o.$$

The transmitted field is thus the same as the incident field, albeit with a 180º phase shift.

b)

$$\langle S_z(z,t)\rangle = \tfrac{1}{2}\mathrm{Re}\{E_x(z,t) H_y^*(z,t)\} = \tfrac{1}{2}\mathrm{Re}\{\{E_x^{(1)} \exp[\mathrm{i}(nk_o z - \omega_o t)] + E_x^{(2)} \exp[\mathrm{i}(-nk_o z - \omega_o t)]\}$$
$$\times \{H_y^{(1)} \exp[-\mathrm{i}(nk_o z - \omega_o t)] + H_y^{(2)} \exp[-\mathrm{i}(-nk_o z - \omega_o t)]\}\}$$
$$= \tfrac{1}{2}\mathrm{Re}\{E_x^{(1)} H_y^{(1)} + E_x^{(2)} H_y^{(2)} + E_x^{(1)} H_y^{(2)} \exp(2\mathrm{i} n k_o z) + E_x^{(2)} H_y^{(1)} \exp(-2\mathrm{i} n k_o z)\}$$
$$= \{(n+1)^2 - (n-1)^2 - 2(n^2-1)\,\mathrm{Re}[\mathrm{i}\sin(2nk_o z)]\}\, E_x^{(\mathrm{i})2}/(8nZ_o) = \tfrac{1}{2} E_x^{(\mathrm{i})2}/Z_o.$$

The final result is obviously the same as the time-averaged rate of energy flow (per unit area per unit time) in the incidence medium as well as that in the transmission medium.

c) The above results will remain essentially the same if $d = m\lambda_o/(2n)$, where $m \neq 1$ is an arbitrary integer. The only change will be in the phase of the transmitted beam when m happens to be *even*, in which case $E_x^{(\mathrm{t})} = E_x^{(\mathrm{i})}$ and $H_y^{(\mathrm{t})} = E_x^{(\mathrm{i})}/Z_o$.