Problem 7.65)

a) $\rho_{12} = (n_1 - n_2)/(n_1 + n_2)$, $\tau_{12} = 2n_1/(n_1 + n_2)$.

$\rho_{21} = (n_2 - n_1)/(n_2 + n_1) = -\rho_{12}$, $\tau_{21} = 2n_2/(n_2 + n_1)$.

$\rho_{23} = (n_2 - n_3)/(n_2 + n_3) = (n_2 - n - \mathrm{i}\kappa)/(n_2 + n + \mathrm{i}\kappa)$, $\tau_{23} = 2n_2/(n_2 + n_3)$.

b) Immediately beneath the entrance facet, $E_0^{(\mathrm{a})}$ receives a contribution from $E_0^{(\mathrm{i})}$ via the transmission coefficient τ_{12}. A second contribution comes from $E_0^{(\mathrm{b})}$ upon reflection at the upper dielectric surface (reflection coefficient = ρ_{21}). However, $E_0^{(\mathrm{b})}$ itself is obtained from $E_0^{(\mathrm{a})}$ after a downward propagation through the thickness d, reflection at the substrate interface (reflection coefficient $= \rho_{23}$), and an upward propagation, again through the thickness d. The self-consistency equation for $E_0^{(\mathrm{a})}$ may thus be written as follows:

$$E_0^{(\mathrm{a})} = \tau_{12}E_0^{(\mathrm{i})} + \rho_{21}\rho_{23}\exp(2\mathrm{i}n_2k_0d)\,E_0^{(\mathrm{a})} \quad \rightarrow \quad E_0^{(\mathrm{a})} = \frac{\tau_{12}}{1 - \rho_{21}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)}E_0^{(\mathrm{i})}.$$

c) The E-field amplitude transmitted into the substrate is obtained by propagating $E_0^{(\mathrm{a})}$ downward through the thickness d, then multiplying by τ_{23} to account for transmission from immediately above to immediately below the dielectric-substrate interface. We will have

$$E_0^{(\mathrm{t})} = \tau_{23}\exp(\mathrm{i}n_2k_0d)\,E_0^{(\mathrm{a})} = \frac{\tau_{12}\tau_{23}\exp(\mathrm{i}2\pi n_2 d/\lambda_0)}{1 - \rho_{21}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)}E_0^{(\mathrm{i})}.$$

d) The reflected E-field amplitude at the top of the dielectric layer has two contributions. The first comes from direct reflection from the top facet of the incident amplitude $E_0^{(\mathrm{i})}$. The second contribution comes from $E_0^{(\mathrm{b})}$ after multiplication by τ_{21}. However, $E_0^{(\mathrm{b})}$ itself arises from the propagation of $E_0^{(\mathrm{a})}$ downward through the thickness d, reflection at the substrate interface, then upward propagation through the thickness d of the dielectric layer. We will have

$$E_0^{(\mathrm{r})} = \rho_{12}E_0^{(\mathrm{i})} + \tau_{21}\rho_{23}\exp(2\mathrm{i}n_2k_0d)\,E_0^{(\mathrm{a})}$$
$$= \left[\rho_{12} + \frac{\tau_{12}\tau_{21}\rho_{23}\exp(2\mathrm{i}n_2k_0d)}{1 - \rho_{21}\rho_{23}\exp(2\mathrm{i}n_2k_0d)}\right]E_0^{(\mathrm{i})} = \left[\frac{\rho_{12} + (\tau_{12}\tau_{21} - \rho_{12}\rho_{21})\rho_{23}\exp(2\mathrm{i}n_2k_0d)}{1 - \rho_{21}\rho_{23}\exp(2\mathrm{i}n_2k_0d)}\right]E_0^{(\mathrm{i})}.$$

Now, using the expressions for $\rho_{12}, \tau_{12}, \rho_{21}, \tau_{21}$ obtained in part (a), we write

$$\tau_{12}\tau_{21} - \rho_{12}\rho_{21} = \frac{4n_1n_2}{(n_1+n_2)^2} + \frac{(n_1-n_2)^2}{(n_1+n_2)^2} = 1.0$$

Consequently,

$$E_0^{(\mathrm{r})} = \frac{\rho_{12} + \rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)}{1 + \rho_{12}\rho_{23}\exp(\mathrm{i}4\pi n_2 d/\lambda_0)}E_0^{(\mathrm{i})}.$$

For a given refractive index n_2, the thickness d of the dielectric layer can be adjusted to control the reflectance of the bare substrate.

e) When $d = m\lambda_0/(2n_2)$, the phase-factor $\exp(\mathrm{i}4\pi n_2 d/\lambda_0)$ appearing in the preceding equation becomes equal to 1.0. We will then have

$$E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})} = \frac{\rho_{12} + \rho_{23}}{1 + \rho_{12}\rho_{23}} = \frac{\left(\frac{n_1-n_2}{n_1+n_2}\right) + \left(\frac{n_2-n_3}{n_2+n_3}\right)}{1+\left(\frac{n_1-n_2}{n_1+n_2}\right)\left(\frac{n_2-n_3}{n_2+n_3}\right)} = \frac{(n_1-n_2)(n_2+n_3)+(n_2-n_3)(n_1+n_2)}{(n_1+n_2)(n_2+n_3)+(n_1-n_2)(n_2-n_3)}$$

$$= \frac{n_1n_2+n_1n_3-n_2^2-n_2n_3+n_1n_2+n_2^2-n_1n_3-n_2n_3}{n_1n_2+n_1n_3+n_2^2+n_2n_3+n_1n_2-n_1n_3-n_2^2+n_2n_3} = \frac{2n_1n_2-2n_2n_3}{2n_1n_2+2n_2n_3} = \frac{n_1 - n_3}{n_1 + n_3}.$$

Clearly, the overall reflection coefficient $E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})}$ in this case is independent of n_2, having the value it would have if the beam was directly incident from free space onto the substrate.