Problem 7.63)

a) $$\boldsymbol{k} = k_\rho\hat{\boldsymbol{\rho}} + k_z\hat{\boldsymbol{z}} = (\omega/c)\sin(\theta_0)\,\hat{\boldsymbol{\rho}} + (\omega/c)\cos(\theta_0)\,\hat{\boldsymbol{z}}. \tag{1}$$

b) $$E_\rho(\boldsymbol{r},t) = -E_0\cos\theta_0\cos(\varphi-\varphi_0)\exp\{\mathrm{i}[k_\rho\rho\cos(\varphi-\varphi_0)+k_z z-\omega t]\}, \tag{2a}$$

$$E_\varphi(\boldsymbol{r},t) = E_0\cos\theta_0\sin(\varphi-\varphi_0)\exp\{\mathrm{i}[k_\rho\rho\cos(\varphi-\varphi_0)+k_z z-\omega t]\}, \tag{2b}$$

$$E_z(\boldsymbol{r},t) = E_0\sin\theta_0\exp\{\mathrm{i}[k_\rho\rho\cos(\varphi-\varphi_0)+k_z z-\omega t]\}. \tag{2c}$$

$$H_\rho(\boldsymbol{r},t) = -(E_0/Z_0)\sin(\varphi-\varphi_0)\exp\{\mathrm{i}[k_\rho\rho\cos(\varphi-\varphi_0)+k_z z-\omega t]\}, \tag{3a}$$

$$H_\varphi(\boldsymbol{r},t) = -(E_0/Z_0)\cos(\varphi-\varphi_0)\exp\{\mathrm{i}[k_\rho\rho\cos(\varphi-\varphi_0)+k_z z-\omega t]\}, \tag{3b}$$

$$H_z(\boldsymbol{r},t) = 0. \tag{3c}$$

c) The following integrals will be useful in subsequent derivations:

i) $$\int_0^{2\pi}\sin\varphi\exp(\mathrm{i}x\cos\varphi)\,d\varphi = -\left(\frac{1}{\mathrm{i}x}\right)\exp(\mathrm{i}x\cos\varphi)\Big|_{\varphi=0}^{2\pi} = -\frac{\exp(\mathrm{i}x)-\exp(\mathrm{i}x)}{\mathrm{i}x} = 0. \tag{4}$$

ii) $$\frac{d}{dx}\int_0^{2\pi}\exp(\mathrm{i}x\cos\varphi)\,d\varphi = \int_0^{2\pi}\mathrm{i}\cos\varphi\exp(\mathrm{i}x\cos\varphi)\,d\varphi = 2\pi J_0'(x) = -2\pi J_1(x)$$

$$\rightarrow \quad \int_0^{2\pi}\cos\varphi\exp(\mathrm{i}x\cos\varphi)\,d\varphi = \mathrm{i}2\pi J_1(x). \tag{5}$$

To find the (E_ρ, E_φ, E_z) and (H_ρ, H_φ, H_z) of the superposition, we integrate the fields obtained in part (b) over φ_0 from 0 to 2π. We find

$$E_\rho^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}E_\rho(\boldsymbol{r},t)d\varphi_0 = -E_0\cos\theta_0\exp[\mathrm{i}(k_z z-\omega t)]\int_0^{2\pi}\cos\varphi\exp(\mathrm{i}k_\rho\rho\cos\varphi)\,d\varphi$$
$$= -\mathrm{i}2\pi E_0\cos\theta_0 J_1(k_\rho\rho)\exp[\mathrm{i}(k_z z-\omega t)], \tag{6a}$$

$$E_\varphi^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}E_\varphi(\boldsymbol{r},t)d\varphi_0 = -E_0\cos\theta_0\exp[\mathrm{i}(k_z z-\omega t)]\int_0^{2\pi}\sin\varphi\exp(\mathrm{i}k_\rho\rho\cos\varphi)\,d\varphi = 0, \tag{6b}$$

$$E_z^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}E_z(\boldsymbol{r},t)d\varphi_0 = E_0\sin\theta_0\exp[\mathrm{i}(k_z z-\omega t)]\int_0^{2\pi}\exp(\mathrm{i}k_\rho\rho\cos\varphi)\,d\varphi$$
$$= 2\pi E_0\sin\theta_0 J_0(k_\rho\rho)\exp[\mathrm{i}(k_z z-\omega t)]. \tag{6c}$$

$$H_\rho^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}H_\rho(\boldsymbol{r},t)d\varphi_0 = (E_0/Z_0)\exp[\mathrm{i}(k_z z-\omega t)]\int_0^{2\pi}\sin\varphi\exp(\mathrm{i}k_\rho\rho\cos\varphi)\,d\varphi = 0, \tag{7a}$$

$$H_\varphi^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}H_\varphi(\boldsymbol{r},t)d\varphi_0 = -(E_0/Z_0)\exp[\mathrm{i}(k_z z-\omega t)]\int_0^{2\pi}\cos\varphi\exp(\mathrm{i}k_\rho\rho\cos\varphi)\,d\varphi$$
$$= -\mathrm{i}2\pi(E_0/Z_0)J_1(k_\rho\rho)\exp[\mathrm{i}(k_z z-\omega t)], \tag{7b}$$

$$H_z^{(\text{total})} = \int_{\varphi_0=0}^{2\pi}H_z(\boldsymbol{r},t)d\varphi_0 = 0. \tag{7c}$$