Problem 7.61)

a) For the transmitted beam, the continuity of k_x yields $k_x^{(\mathrm{t})} = k_x^{(\mathrm{i})} = (\omega/c)n_0 \sin\theta$. Also, the E-field amplitude immediately beneath the interface will be $\boldsymbol{E}_{\mathrm{s}}^{(\mathrm{t})} = \tau_{\mathrm{s}} E_{\mathrm{s}}^{(\mathrm{i})} \hat{\boldsymbol{y}}$. Thus,

$$\boldsymbol{k}^{(\mathrm{t})} = k_x \hat{\boldsymbol{x}} + k_z^{(\mathrm{t})} \hat{\boldsymbol{z}} = (\omega/c)\left[n_0 \sin\theta\, \hat{\boldsymbol{x}} - \sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2 \sin^2\theta}\, \hat{\boldsymbol{z}}\right].$$

$$\boldsymbol{E}^{(\mathrm{t})}(\boldsymbol{r},t) = \tau_s E_s^{(\mathrm{i})} \exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{r} - \omega t\right)\right] \hat{\boldsymbol{y}}.$$

The square root must be chosen such that the imaginary part of $k_z^{(\mathrm{t})}$ is *negative*, so that the field amplitude will decay exponentially as $z \to -\infty$.

From Maxwell's 3rd equation: $\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t \quad \to \quad \boldsymbol{k}^{(\mathrm{t})} \times \tau_s \boldsymbol{E}_s^{(\mathrm{i})} = \mu_0 \omega \boldsymbol{H}_0^{(\mathrm{t})}$

$$\to \quad \boldsymbol{H}_0^{(\mathrm{t})} = Z_0^{-1}\left[n_0 \sin\theta\, \hat{\boldsymbol{x}} - \sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2 \sin^2\theta}\, \hat{\boldsymbol{z}}\right] \times \tau_{\mathrm{s}} E_{\mathrm{s}}^{(\mathrm{i})} \hat{\boldsymbol{y}}$$

$$= Z_0^{-1} \tau_{\mathrm{s}} E_{\mathrm{s}}^{(\mathrm{i})} \left[\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2 \sin^2\theta}\, \hat{\boldsymbol{x}} + n_0 \sin\theta\, \hat{\boldsymbol{z}}\right].$$

Consequently, $\boldsymbol{H}^{(\mathrm{t})}(\boldsymbol{r},t) = \boldsymbol{H}_0^{(\mathrm{t})} \exp\left[\mathrm{i}\left(\boldsymbol{k}^{(\mathrm{t})} \cdot \boldsymbol{r} - \omega t\right)\right]$.

b) $\langle \boldsymbol{S}(\boldsymbol{r},t) \rangle = ½\mathrm{Re}\{\boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}^*(\boldsymbol{r},t)\}$

$$= ½\mathrm{Re}\Big\{\tau_s E_s^{(\mathrm{i})} \exp[\mathrm{i}(k_x x + k_z^{(\mathrm{t})} z)]\, \hat{\boldsymbol{y}}$$

$$\times Z_0^{-1} \tau_{\mathrm{s}}^* E_{\mathrm{s}}^{*(\mathrm{i})} \left[\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2 \sin^2\theta}^{\,*}\, \hat{\boldsymbol{x}} + n_0 \sin\theta\, \hat{\boldsymbol{z}}\right] \exp[-\mathrm{i}(k_x x + k_z^{*(\mathrm{t})} z)]\Big\}$$

$$= ½ Z_0^{-1} \left|\tau_s E_s^{(\mathrm{i})}\right|^2 \left[n_0 \sin\theta\, \hat{\boldsymbol{x}} - \mathrm{Re}\sqrt{(n+\mathrm{i}\kappa)^2 - n_0^2 \sin^2\theta}\, \hat{\boldsymbol{z}}\right] \exp\left\{-2\mathrm{Im}[k_z^{(\mathrm{t})}]z\right\}.$$

As pointed out earlier, $\mathrm{Im}[k_z^{(\mathrm{t})}]$ is negative and, therefore, $\langle \boldsymbol{S}(\boldsymbol{r},t) \rangle$ decays exponentially as $z \to -\infty$.