Problem 7.55) Since the incident beam is circularly polarized, its p- and s-components are equal in magnitude and 90° apart in phase. Considering that $\theta = 45°$, we have $\sin\theta = \cos\theta = 1/\sqrt{2}$.

a) $$\rho_p = \frac{\sqrt{n^2-\sin^2\theta}-n^2\cos\theta}{\sqrt{n^2-\sin^2\theta}+n^2\cos\theta} = \frac{\sqrt{1.5^2-½}-1.5^2/\sqrt{2}}{\sqrt{1.5^2-½}+1.5^2/\sqrt{2}} \cong \frac{1.323-1.591}{1.323+1.591} \cong -0.092,$$

$$\rho_s = \frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{\cos\theta+\sqrt{n^2-\sin^2\theta}} = \frac{(1/\sqrt{2})-\sqrt{1.5^2-½}}{(1/\sqrt{2})+\sqrt{1.5^2-½}} \cong \frac{0.707-1.323}{0.707+1.323} \cong -0.303.$$

The reflectivity of the dielectric surface for the incident (circularly-polarized) beam is thus given by $R = ½(|\rho_p|^2 + |\rho_s|^2) \cong 0.05$.

b) The polarization state of the reflected beam is elliptical, because the reflected p- and s-components continue to have a 90° phase difference (i.e., same as the incident beam), but the two amplitudes are no longer equal: $E_s^{(r)}/E_p^{(r)} = 0.303/0.092 \cong 3.29$. The sense of rotation of the E-field around the ellipse is the reverse of that of the incident beam, so that a right-circularly-polarized incident beam will result in a left-elliptically-polarized reflected beam, and vice-versa.

c) The Fresnel transmission coefficients are readily computed as follows:

$$\tau_p = \frac{2\sqrt{n^2-\sin^2\theta}}{\sqrt{n^2-\sin^2\theta}+n^2\cos\theta} = \frac{2\sqrt{1.5^2-½}}{\sqrt{1.5^2-½}+1.5^2/\sqrt{2}} \cong \frac{2\times1.323}{1.323+1.591} \cong 0.908,$$

$$\tau_s = \frac{2\cos\theta}{\cos\theta+\sqrt{n^2-\sin^2\theta}} = \frac{\sqrt{2}}{(1/\sqrt{2})+\sqrt{1.5^2-½}} \cong \frac{1.414}{0.707+1.323} \cong 0.697.$$

d) Since $\tau_s = E_{y0}^{(t)}/E_{y0}^{(i)} = E_s^{(t)}/E_s^{(i)}$, the s-component of the transmitted E-field amplitude is $\tau_s = 0.697$ times the s-component of the incident E-field amplitude. However, with the p-component we have $\tau_p = E_{x0}^{(t)}/E_{x0}^{(i)} = (E_p^{(t)}\cos\theta')/(E_p^{(i)}\cos\theta)$. Using Snell's law, $\sin\theta = n\sin\theta'$, we find $\theta' = 28.126°$. Therefore, $\cos\theta\,/\cos\theta' \cong 0.802$, and $E_p^{(t)}/E_p^{(i)} \cong 0.728$. As was the case with the reflected beam, we see that the transmitted p- and s-components have unequal magnitudes: $E_s^{(t)}/E_p^{(t)} \cong 0.697/0.728 \cong 0.957$. The phase difference between $E_s^{(t)}$ and $E_p^{(t)}$ is still 90°, which is the phase difference between the s- and p-components of the incident wave. We conclude that the transmitted beam is elliptically polarized, albeit not too far from circular, having the same sense of rotation of the E-field around the ellipse as that of the incident beam.

e) The magnitude of the time-averaged Poynting vector is $\langle S\rangle = ½n|E_0|^2/Z_0$, where Z_0 is the impedance of free space. This means that the Poynting vector of the transmitted p-light is $1.5\times0.728^2 = 0.795$ times that of the incident beam. Similarly, the Poynting vector of the transmitted s-light is $1.5\times0.697^2 = 0.729$ times that of the incident beam. The total Poynting vector (i.e., p plus s) of the transmitted beam is, therefore, $½(0.795+0.729) \cong 0.762$ times that of the incident beam.

f) In contrast to the reflected beam, which has the same cross-sectional area as the incident beam, the cross-sectional area of the transmitted beam is greater than that of the incident beam by $\cos\theta'/\cos\theta \cong 1.247$. The transmitted optical power is, therefore, $0.762\times1.247 \cong 0.95$ times the incident optical power. In part (a) we found the reflected optical power to be 0.05 times the incident power. Conservation of energy is thus confirmed.