Problem 47)

a) Denoting the wave-number by $k_0 = \omega/c$, and the normalized k-vector by $\boldsymbol{\sigma} = \boldsymbol{k}/k_0$, we write

$$\boldsymbol{E}(\boldsymbol{r},t) = E_0\hat{\boldsymbol{x}}\exp\{\mathrm{i}k_0[n(\omega)z - ct]\}.$$

$$Z_0\boldsymbol{H}_0 = \boldsymbol{\sigma}\times\boldsymbol{E}_0 \quad\rightarrow\quad Z_0\boldsymbol{H}_0 = n(\omega)E_0(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{x}}) \quad\rightarrow\quad \boldsymbol{H}_0 = n(\omega)E_0\hat{\boldsymbol{y}}/Z_0$$

$$\rightarrow\quad \boldsymbol{H}(\boldsymbol{r},t) = \left[\frac{n(\omega)E_0}{Z_0}\right]\hat{\boldsymbol{y}}\exp\{\mathrm{i}k_0[n(\omega)z - ct]\}.$$

b) $n(\omega) = \sqrt{\varepsilon(\omega)} \quad\rightarrow\quad \varepsilon(\omega) = n^2(\omega).$

$\varepsilon(\omega) = 1 + \chi(\omega) \quad\rightarrow\quad \chi(\omega) = n^2(\omega) - 1.$

c) $\boldsymbol{P}(\boldsymbol{r},t) = \varepsilon_0\chi(\omega)E_0\hat{\boldsymbol{x}}\exp\{\mathrm{i}k_0[n(\omega)z - ct]\}$

$$= \varepsilon_0[n^2(\omega) - 1]E_0\hat{\boldsymbol{x}}\exp\{\mathrm{i}k_0[n(\omega)z - ct]\}.$$

$$\rho_{\text{bound}}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\cdot\boldsymbol{P}(\boldsymbol{r},t) = -\left(\frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}\right) = 0.$$

$$\boldsymbol{J}_{\text{bound}}(\boldsymbol{r},t) = \frac{\partial\boldsymbol{P}(\boldsymbol{r},t)}{\partial t} = -\mathrm{i}\omega\varepsilon_0[n^2(\omega) - 1]E_0\hat{\boldsymbol{x}}\exp\{\mathrm{i}k_0[n(\omega)z - ct]\}.$$

The actual $\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{P}, \boldsymbol{J}_{\text{bound}}$ are, of course, given by the *real parts* of the above expressions.