Problem 7.46)

a) $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}=1 \quad\rightarrow\quad (\mathrm{i}\sigma_x\hat{\boldsymbol{x}}+\sigma_z\hat{\boldsymbol{z}})\cdot(\mathrm{i}\sigma_x\hat{\boldsymbol{x}}+\sigma_z\hat{\boldsymbol{z}})=-\sigma_x^2+\sigma_z^2=1 \quad\rightarrow\quad \sigma_z^2=1+\sigma_x^2.$

b) $\boldsymbol{\nabla}\cdot\boldsymbol{E}=0 \quad\rightarrow\quad \boldsymbol{\sigma}\cdot\boldsymbol{E}_0=0 \quad\rightarrow\quad \mathrm{i}\sigma_x E_{x0}+\sigma_z E_{z0}=0.$

c) $\boldsymbol{\nabla}\cdot\boldsymbol{B}=0 \quad\rightarrow\quad \boldsymbol{\sigma}\cdot\boldsymbol{H}_0=0 \quad\rightarrow\quad \mathrm{i}\sigma_x H_{x0}+\sigma_z H_{z0}=0.$

d) $\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t \;\rightarrow\; \mathrm{i}k_0\boldsymbol{\sigma}\times\boldsymbol{E}_0=\mathrm{i}\omega\mu_0\boldsymbol{H}_0 \rightarrow (\mathrm{i}\sigma_x\hat{\boldsymbol{x}}+\sigma_z\hat{\boldsymbol{z}})\times(E_{x0}\hat{\boldsymbol{x}}+E_{y0}\hat{\boldsymbol{y}}+E_{z0}\hat{\boldsymbol{z}})=Z_0\boldsymbol{H}_0$

$\rightarrow\quad \mathrm{i}\sigma_x E_{y0}\hat{\boldsymbol{z}}+(\sigma_z E_{x0}-\mathrm{i}\sigma_x E_{z0})\hat{\boldsymbol{y}}-\sigma_z E_{y0}\hat{\boldsymbol{x}}=Z_0\boldsymbol{H}_0$

$\rightarrow\quad Z_0 H_{0x}=-\sigma_z E_{y0};\qquad Z_0 H_{0y}=\sigma_z E_{x0}-\mathrm{i}\sigma_x E_{z0};\qquad Z_0 H_{0z}=\mathrm{i}\sigma_x E_{y0}.$

e) If $E_{z0}=0$, then from (b) we have $E_{x0}=0$, and from (d) we find $H_{y0}=0$.

f) If $H_{z0}=0$, then from (c) we have $H_{x0}=0$, and from (d) we find $E_{y0}=0$.