a) $\sigma_x'' = \sin\theta$, $\sigma_y'' = 0$, $\sigma_z'' = \sqrt{\epsilon(\omega) - \sin^2\theta} = \left[(\epsilon_R(\omega) - \sin^2\theta) + i\epsilon_I(\omega)\right]^{1/2} \Rightarrow$

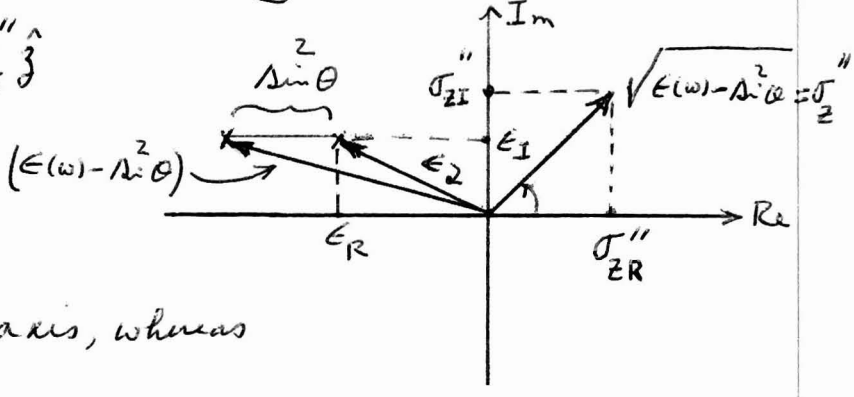
$$\vec{\sigma}'' = (\sin\theta)\,\hat{x} + \left[(\epsilon_R(\omega) - \sin^2\theta) + i\epsilon_I(\omega)\right]^{1/2}\hat{z}$$

$\uparrow\ \sigma_{zR}'' + i\sigma_{zI}''$

b) When $\theta = 0^\circ$, the x-component of $\vec{\sigma}''$ is zero (i.e., $\sin\theta = 0$). Then $\vec{\sigma}''$ will only have a z-component, namely, $\sigma_z'' = \sqrt{\epsilon_R(\omega) + i\epsilon_I(\omega)}$. Therefore, both $\vec{\sigma}_R''$ and $\vec{\sigma}_I''$ are parallel to the z-axis, which makes $\vec{\sigma}_R'' \parallel \vec{\sigma}_I''$.

c) $\vec{\sigma}'' = (\sin\theta\,\hat{x} + \sigma_{zR}''\hat{z}) + i\sigma_{zI}''\hat{z}$

$$\Rightarrow \begin{cases} \vec{\sigma}_R'' = \sin\theta\,\hat{x} + \sigma_{zR}''\hat{z} \\ \vec{\sigma}_I'' = \sigma_{zI}''\hat{z} \end{cases}$$



Clearly, $\vec{\sigma}_I''$ is along the z-axis, whereas $\vec{\sigma}_R''$ has a component along $\hat{x}$ and another component along $\hat{z}$. The angle between $\vec{\sigma}_R''$ and $\vec{\sigma}_I''$ can therefore be anything between 0° and 90°.

d) When $\epsilon_I(\omega) = 0$, two possibilities exist: either $\epsilon_R(\omega) \geq \sin^2\theta$ or $\epsilon_R(\omega) < \sin^2\theta$.

Case I) $\epsilon_R(\omega) - \sin^2\theta \geq 0 \Rightarrow \vec{\sigma}_R'' = (\sin\theta)\,\hat{x} + \sqrt{\epsilon_R(\omega) - \sin^2\theta}\,\hat{z}$; $\vec{\sigma}_I'' = 0$

Case II) $\epsilon_R(\omega) - \sin^2\theta < 0 \Rightarrow \vec{\sigma}_R'' = (\sin\theta)\,\hat{x}$; $\vec{\sigma}_I'' = \sqrt{\sin^2\theta - \epsilon_R(\omega)}\,\hat{z}$

In Case II, note that $\vec{\sigma}_R'' \perp \vec{\sigma}_I''$. Therefore, $\vec{\sigma}_R''$ is orthogonal to $\vec{\sigma}_I''$ when $\epsilon_R(\omega) < 0$ or when $\sin^2\theta > \epsilon_R(\omega)$, provided that $\epsilon_I(\omega) = 0$ in either case.