Problem 38)

a) Case of s-polarization,

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \xrightarrow[n\to\infty]{} \frac{\cos\theta - n}{\cos\theta + n} \longrightarrow \frac{-n}{+n} = -1 \checkmark$$

Incident: $\sigma_x = \sin\theta$, $\sigma_y = 0$, $\sigma_z = -\cos\theta$; $\vec{E}_s = E_y \hat{y}$; $Z_0 \vec{H}_0 = \vec{\sigma} \times \vec{E}_s = E_y(\cos\theta\hat{x} + \sin\theta\hat{z})$

Reflected: $\sigma'_x = \sin\theta$, $\sigma'_y = 0$, $\sigma'_z = +\cos\theta$; $\vec{E}'_s = r_s E_y \hat{y} = -E_y \hat{y}$; $Z_0 \vec{H}'_0 = \vec{\sigma}' \times \vec{E}'_s = E_y(\cos\theta\hat{x} - \sin\theta\hat{z})$

Continuity of $\vec{E}_\parallel$ at the surface: $\vec{E}_s + \vec{E}'_s = E_y\hat{y} + r_s E_y \hat{y} = (E_y - E_y)\hat{y} = 0$.

The $\vec{E}_{\parallel}$-field is zero in the air just above the mirror, and also in the metal just beneath the surface. This is not inconsistent with the existence of a surface current, because the conductivity of the mirror is ∞.

Continuity of $\vec{H}_{\parallel}$ at the surface: $H_x\hat{x} + H_x'\hat{x} = \frac{E_y}{Z_0}(\cos\theta + \cos\theta)\hat{x}$ $= \frac{2E_y\cos\theta}{Z_0}\hat{x}$. The spatio-temporal dependence of the fields is $\exp(ik_0\vec{\sigma}\cdot\vec{r} - i\omega t)$. The reflected and incident beams have $\sigma_x = \sigma_x' = \sin\theta$ and, at the mirror surface, $z=0$. The common spatio-temporal factor at the mirror surface is thus $\exp(ik_0\sigma_x x - i\omega t)$. The total magnetic field at the top of the mirror surface is $\frac{2E_s\cos\theta}{Z_0}\exp(ik_0\sin\theta\, x - i\omega t)\hat{x}$. The $\vec{H}$-field beneath the surface is zero (Perfect Conductor). The discontinuity of the H-field is then equal to the surface current density, that is,

$$\vec{J}_s(\vec{r},t) = \frac{2E_s\cos\theta}{Z_0}e^{ik_0(\sin\theta\, x - ct)}\hat{y}$$

← Use $\vec{\nabla}\times\vec{H} = \vec{J}$ to see that $\vec{J}_s$ is along $\hat{y}$.

There is no $\perp$ $\vec{E}$-field at the surface, therefore, $\sigma_s(\vec{r},t) = 0$. This can also be inferred from $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$, because $\vec{\nabla}\cdot\vec{J}_s = 0$.

b) Case of p-polarization: $r_p = \frac{\sqrt{n^2-\sin^2\theta} - n^2\cos\theta}{\sqrt{n^2-\sin^2\theta} + n^2\cos\theta} \xrightarrow[n\to\infty]{} \frac{n - n^2\cos\theta}{n + n^2\cos\theta} \to \frac{-n^2\cos\theta}{n^2\cos\theta} = -1$ ✓

Incident: $\sigma_x = \sin\theta$, $\sigma_y = 0$, $\sigma_z = -\cos\theta$; $\vec{E}_p = E_p(\cos\theta\hat{x} + \sin\theta\hat{z})$, $Z_0\vec{H}_0 = \vec{\sigma}\times\vec{E}_p = -E_p\hat{y}$

Reflected: $\sigma_x' = \sin\theta$, $\sigma_y' = 0$, $\sigma_z' = +\cos\theta$; $\vec{E}_p' = -E_p(\cos\theta\hat{x} - \sin\theta\hat{z})$; $Z_0\vec{H}_0' = \vec{\sigma}'\times\vec{E}_p' = -E_p\hat{y}$

$\vec{H}$-field at the top surface of mirror $= -\frac{2E_p}{Z_0}\exp(ik_0\sin\theta\, x - i\omega t)\hat{y} \Rightarrow$

$$\vec{J}_s(\vec{r},t) = \frac{2E_p}{Z_0}\exp[ik_0(\sin\theta\, x - ct)]\,\hat{x}$$

← use $\vec{\nabla}\times\vec{H} = \vec{J}$ to see that $\vec{J}_s$ is along $\hat{x}$.

$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 \Rightarrow \sigma_s(\vec{r},t) = \epsilon_0(E_\perp^{\text{above}} - E_\perp^{\text{below}}) = \epsilon_0 E_z^{\text{above}} = 2\epsilon_0 E_p \sin\theta \exp[ik_0(\sin\theta\, x - ct)]$

It is readily verified that $\vec{\nabla}\cdot\vec{J}_s + \frac{\partial}{\partial t}\sigma_s = 0$.