Problem 36)

a) $\vec{E}_1(\vec{r},t) = E_{x_1}\hat{x}\exp\{ik_0(y\sin\theta + z\cos\theta - ct)\}$

$\vec{E}_2(\vec{r},t) = E_{x_2}\hat{x}\exp\{ik_0(-y\sin\theta + z\cos\theta - ct)\}$

$$\vec{H}_1(\vec{r},t) = \frac{1}{Z_0}\vec{\sigma}_1 \times \vec{E}_1(\vec{r},t) = \frac{E_{x1}}{Z_0}(\cos\theta\,\hat{y} - \sin\theta\,\hat{z})\exp[ik_0(y\sin\theta + z\cos\theta - ct)]$$

$$\vec{H}_2(\vec{r},t) = \frac{1}{Z_0}\vec{\sigma}_2 \times \vec{E}_2(\vec{r},t) = \frac{E_{x2}}{Z_0}(\cos\theta\,\hat{y} + \sin\theta\,\hat{z})\exp[ik_0(-y\sin\theta + z\cos\theta - ct)]$$

b) $\langle\vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*) = \frac{1}{2}\mathrm{Re}\{(\vec{E}_1+\vec{E}_2)\times(H_1^*+H_2^*)\} =$

$$\frac{1}{2}\mathrm{Re}(\vec{E}_1\times\vec{H}_1^*) + \frac{1}{2}\mathrm{Re}(\vec{E}_2\times\vec{H}_2^*) + \frac{1}{2}\mathrm{Re}(\vec{E}_1\times\vec{H}_2^*) + \frac{1}{2}\mathrm{Re}(\vec{E}_2\times\vec{H}_1^*) =$$

$$\frac{|E_{x1}|^2}{2Z_0}\hat{x}\times(\cos\theta\,\hat{y} - \sin\theta\,\hat{z}) + \frac{|E_{x2}|^2}{2Z_0}\hat{x}\times(\cos\theta\,\hat{y} + \sin\theta\,\hat{z}) +$$

$$\frac{1}{2Z_0}\mathrm{Re}\left\{E_{x1}e^{ik_0(y\sin\theta + z\cos\theta - ct)}E_{x2}^*e^{-ik_0(-y\sin\theta + z\cos\theta - ct)}\right\}\hat{x}\times(\cos\theta\,\hat{y} + \sin\theta\,\hat{z}) +$$

$$\frac{1}{2Z_0}\mathrm{Re}\left\{E_{x2}e^{ik_0(-y\sin\theta + z\cos\theta - ct)}E_{x1}^*e^{-ik_0(y\sin\theta + z\cos\theta - ct)}\right\}\hat{x}\times(\cos\theta\,\hat{y} - \sin\theta\,\hat{z})$$

$$\Rightarrow \langle\vec{S}(\vec{r},t)\rangle = \frac{|E_{x1}|^2}{2Z_0}(\cos\theta\,\hat{z} + \sin\theta\,\hat{y}) + \frac{|E_{x2}|^2}{2Z_0}(\cos\theta\,\hat{z} - \sin\theta\,\hat{y})$$

$$+ \frac{|E_{x1}||E_{x2}|}{2Z_0}\mathrm{Re}\left\{e^{i(2k_0 y\sin\theta + \phi_{x1} - \phi_{x2})}\right\}(\cos\theta\,\hat{z} - \sin\theta\,\hat{y})$$

$$+ \frac{|E_{x1}||E_{x2}|}{2Z_0}\mathrm{Re}\left\{e^{-i(2k_0 y\sin\theta + \phi_{x1} - \phi_{x2})}\right\}(\cos\theta\,\hat{z} + \sin\theta\,\hat{y})$$

$$\Rightarrow \langle\vec{S}(\vec{r},t)\rangle = \frac{|E_{x1}|^2+|E_{x2}|^2}{2Z_0}\cos\theta\,\hat{z} + \frac{|E_{x1}|^2-|E_{x2}|^2}{2Z_0}\sin\theta\,\hat{y} + \frac{|E_{x1}||E_{x2}|}{Z_0}\cos\theta\cos(2k_0\sin\theta\,y + \phi_{x1} - \phi_{x2})\hat{z}$$

The last term in the above expression is the interference term, modulating the Poynting Vector Component $\langle S_z\rangle$ along the y-direction according to the function $\cos(2k_0\sin\theta\,y + \phi_{x1} - \phi_{x2})$. The Component $\langle S_z\rangle$ is thus strong when the fringes are bright and weak when the fringes are dark.