Opti 501 **Solutions**

Problem 30)

$$\vec{\sigma}\cdot\vec{E}_o = 0 \Rightarrow (\vec{\sigma}_R + i\vec{\sigma}_I)\cdot(\vec{E}_{oR} + i\vec{E}_{oI}) = 0 \Rightarrow (\vec{\sigma}_R\cdot\vec{E}_{oR} - \vec{\sigma}_I\cdot\vec{E}_{oI}) + i(\vec{\sigma}_R\cdot\vec{E}_{oI} + \vec{\sigma}_I\cdot\vec{E}_{oR}) = 0$$

$$\Rightarrow \begin{cases} \vec{\sigma}_R\cdot\vec{E}_{oR} - \vec{\sigma}_I\cdot\vec{E}_{oI} = 0 \;\checkmark \\ \vec{\sigma}_R\cdot\vec{E}_{oI} + \vec{\sigma}_I\cdot\vec{E}_{oR} = 0 \;\checkmark \end{cases} \quad \text{For homogeneous plane-wave} \Rightarrow \begin{cases} \vec{\sigma}_R\cdot\vec{E}_{oR} = 0 \;\checkmark \\ \vec{\sigma}_R\cdot\vec{E}_{oI} = 0 \;\checkmark \end{cases}$$

Similar relations exist between $\vec{\sigma}_R, \vec{\sigma}_I$ and $\vec{H}_{oR}, \vec{H}_{oI}$, namely,

$$\begin{cases} \vec{\sigma}_R\cdot\vec{H}_{oR} - \vec{\sigma}_I\cdot\vec{H}_{oI} = 0 \;\checkmark \\ \vec{\sigma}_R\cdot\vec{H}_{oI} + \vec{\sigma}_I\cdot\vec{H}_{oR} = 0 \;\checkmark \end{cases} \Longrightarrow \text{For homogeneous plane-wave} \begin{cases} \vec{\sigma}_R\cdot\vec{H}_{oR} = 0 \;\checkmark \\ \vec{\sigma}_R\cdot\vec{H}_{oI} = 0 \;\checkmark \end{cases}$$

$$\vec{\sigma}\times\vec{E}_o = Z_o\vec{H}_o \Rightarrow (\vec{\sigma}_R + i\vec{\sigma}_I)\times(\vec{E}_{oR} + i\vec{E}_{oI}) = Z_o(\vec{H}_{oR} + i\vec{H}_{oI}) \Rightarrow$$

$$(\vec{\sigma}_R\times\vec{E}_{oR} - \vec{\sigma}_I\times\vec{E}_{oI}) + i(\vec{\sigma}_I\times\vec{E}_{oR} + \vec{\sigma}_R\times\vec{E}_{oI}) = Z_o(\vec{H}_{oR} + i\vec{H}_{oI}) \Rightarrow$$

$$\begin{cases} \vec{\sigma}_R\times\vec{E}_{oR} - \vec{\sigma}_I\times\vec{E}_{oI} = Z_o\vec{H}_{oR} \;\checkmark \\ \vec{\sigma}_I\times\vec{E}_{oR} + \vec{\sigma}_R\times\vec{E}_{oI} = Z_o\vec{H}_{oI} \;\checkmark \end{cases} \Rightarrow \text{For homogeneous plane-wave} \begin{cases} \vec{\sigma}_R\times\vec{E}_{oR} = Z_o\vec{H}_{oR} \;\checkmark \\ \vec{\sigma}_R\times\vec{E}_{oI} = Z_o\vec{H}_{oI} \;\checkmark \end{cases}$$

$$Z_o\vec{H}_o\times\vec{\sigma} = \vec{E}_o \Rightarrow Z_o(\vec{H}_{oR} + i\vec{H}_{oI})\times(\vec{\sigma}_R + i\vec{\sigma}_I) = \vec{E}_{oR} + i\vec{E}_{oI} \Rightarrow$$

$$\begin{cases} Z_o(\vec{H}_{oR}\times\vec{\sigma}_R - \vec{H}_{oI}\times\vec{\sigma}_I) = \vec{E}_{oR} \;\checkmark \\ Z_o(\vec{H}_{oR}\times\vec{\sigma}_I + \vec{H}_{oI}\times\vec{\sigma}_R) = \vec{E}_{oI} \;\checkmark \end{cases} \Rightarrow \text{For homogeneous plane-wave} \begin{cases} Z_o\vec{H}_{oR}\times\vec{\sigma}_R = \vec{E}_{oR} \;\checkmark \\ Z_o\vec{H}_{oI}\times\vec{\sigma}_R = \vec{E}_{oI} \;\checkmark \end{cases}$$

$$\vec{\sigma}\cdot\vec{\sigma} = 1 \Rightarrow (\vec{\sigma}_R + i\vec{\sigma}_I)\cdot(\vec{\sigma}_R + i\vec{\sigma}_I) = 1 \Rightarrow (\vec{\sigma}_R\cdot\vec{\sigma}_R - \vec{\sigma}_I\cdot\vec{\sigma}_I) + i(\vec{\sigma}_R\cdot\vec{\sigma}_I + \vec{\sigma}_I\cdot\vec{\sigma}_R) = 1$$

$$\Rightarrow \begin{cases} \vec{\sigma}_R\cdot\vec{\sigma}_R - \vec{\sigma}_I\cdot\vec{\sigma}_I = 1 \;\checkmark \\ \vec{\sigma}_R\cdot\vec{\sigma}_I = 0. \;\checkmark \end{cases} \Rightarrow \begin{cases} |\vec{\sigma}_R|^2 - |\vec{\sigma}_I|^2 = 1 \;\checkmark \\ \vec{\sigma}_R \perp \vec{\sigma}_I \;\checkmark \end{cases}$$