Opti 501 **Solutions**

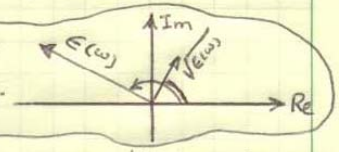
Problem 29)

$\vec{\sigma}\cdot\vec{\sigma} = \epsilon(\omega) \Rightarrow (\vec{\sigma}_R + i\vec{\sigma}_I)\cdot(\vec{\sigma}_R + i\vec{\sigma}_I) = \epsilon_R(\omega) + i\epsilon_I(\omega) \Rightarrow$

$|\vec{\sigma}_R|^2 - |\vec{\sigma}_I|^2 + 2i\,\vec{\sigma}_R\cdot\vec{\sigma}_I = \epsilon_R(\omega) + i\epsilon_I(\omega) = (n_R + in_I)^2 = n_R^2 - n_I^2 + 2in_R n_I \Rightarrow$

$$\begin{cases} |\vec{\sigma}_R|^2 - |\vec{\sigma}_I|^2 = \epsilon_R(\omega) = n_R^2 - n_I^2 \\ \vec{\sigma}_R\cdot\vec{\sigma}_I = \frac{1}{2}\epsilon_I(\omega) = n_R n_I \end{cases}$$

when $\epsilon_I(\omega) \geq 0$, both $n_R \geq 0$, $n_I \geq 0$.



a) $\vec{\sigma}_R \perp \vec{\sigma}_I \Rightarrow \vec{\sigma}_R\cdot\vec{\sigma}_I = 0 \Rightarrow \epsilon_I(\omega) = 0 \Rightarrow$ if $\epsilon_R(\omega) > 0$ then $n_I = 0$; if $\epsilon_R(\omega) < 0$ then $n_R = 0$.

b) $\vec{\sigma}_R \parallel \vec{\sigma}_I \Rightarrow \vec{\sigma}_R\cdot\vec{\sigma}_I = |\vec{\sigma}_R|\,|\vec{\sigma}_I| = n_R n_I \Rightarrow |\vec{\sigma}_I| = \frac{n_R n_I}{|\vec{\sigma}_R|} \Rightarrow |\vec{\sigma}_R|^2 - \frac{n_R^2 n_I^2}{|\vec{\sigma}_R|^2} = n_R^2 - n_I^2 \Rightarrow |\vec{\sigma}_R| = n_R,\ |\vec{\sigma}_I| = n_I.$